UNIVERSITY OF SURREY©

B. Sc. Undergraduate Programmes in Computing

Level HE2 Examination

Module MAT 2016 Computational Operations Research

Time allowed – 2 hrs

Autumn Semester 2008/09

Attempt THREE questions.

If a candidate attempts more than THREE questions
only the best THREE questions will be taken into account.

Question 1

A refinery produces two types of lubricating oil, type A and type B. To produce 1000 litres of type A oil requires two hours in process 1, two hours in process 2 and six hours in process 3. To produce 1000 litres of type B oil requires four hours in process 1, two hours in process 2 and three hours in process 3. The availability of the process 1 plant is twelve hours per day, that of process 2 is seven hours per day and that of process 3 twenty hours per day. The profit on 1000 litres of Type A is £40 and on Type B £50.

The ompany wants to know how much of each type of oil to produce in order to maximise their profit per day, assuming that all the production can be sold.

(a) What are the decision variables for this problem? [2]

(b) What is the objective function? [2]

(c) State the constraints as inequalities. [3]

(d) Sketch the feasible region, indicating the coordinates of each of the corner points. On your diagram, show the contour associated with a daily profit of £200. [4]

(e) Using this contour or otherwise (but NOT by using the Simplex Method), determine the optimum solution to the problem, justifying your approach. [4]

(f) How much of each type of oil should the Company produce each day? [2]

(g) The demand for Type A oil has risen greatly and the Company intends to increase its price. By how much can the profit on Type A oil be increased before the Company should produce this type of oil only? How much profit would the company make at this profit level. [3]

(h) By reorganising the workforce it is possible to increase the number of hours that the plant for process 2 is available. In order to maximise profit, how many extra hours should process 2 be made available? (carefully explain your reasoning) How much of each type of oil will the refinery then produce and what will be the company's profit? [5]

Question 2

(a) Put the following problem in standard form. Do NOT attempt to solve it.

Minimize

$$z = 4x_1 - 7x_2 + 3x_3$$

subject to the constraints

$$\begin{aligned} 3x_1 - 4x_2 + x_3 &\geq 12 \\ x_1 + 2x_2 - 3x_3 &\leq -4 \\ x_1 + x_2 + x_3 &= 2 \end{aligned}$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0$$

[10]

(b) Consider the following problem:

Maximize

$$z = 4x_1 + 5x_2$$

subject to the constraints

$$\begin{aligned} 5x_1 + 2x_2 &\leq 12 \\ 3x_1 + 4x_2 &\leq 10 \end{aligned}$$

and

$$x_1 \geq 0, \quad x_2 \geq 0$$

(i) Write this problem in standard form. [2]

(ii) Find an initial basic feasible solution, stating which are the basic variables and which are the non-basic variables. [2]

(iii) Construct the initial tableau for the Simplex Method. [2]

(iv) Use the tableau form of the Simplex Method to find the optimal solution of the problem. At each stage, state the entering and departing variables from the basis and explain your reasoning. [8]

(v) From the optimal tableau, determine the optimal value of z and state the values of all the variables at this solution. [1]

Question 3

(a) Consider the linear programming problem

maximise profit, given by
$z = 3x_1 + 2x_2$

subject to these constraints

$x_1 + x_2 \leq 6$ (i)
$2x_1 + 3x_2 \leq 12$ (ii)
$3x_1 + x_2 \leq 12$ (iii)

$x_1 \geq 0, \quad x_2 \geq 0.$

The optimum simplex tableau for this problem is as follows:

Basic	z	x_1	x_2	x_3	x_4	x_5	Solution
x_3	0	0	0	1	$-\frac{2}{3}$	$-\frac{1}{7}$	$\frac{6}{7}$
x_2	0	0	1	0	$\frac{3}{7}$	$-\frac{2}{7}$	$1\frac{5}{7}$
x_1	0	1	0	0	$-\frac{1}{7}$	$\frac{3}{7}$	$3\frac{3}{7}$
z	1	0	0	0	$\frac{3}{7}$	$\frac{5}{7}$	$13\frac{5}{7}$

An additional variable x_6 is now to be added to the problem, requiring 1 unit of the resources represented by constraint (i), 2 units of the resources represented by constraint (ii) and 1 unit of the resources represented by constraint (iii) per unit of x_6. The profit per unit of x_6 is p.

(i) Express the new problem in terms of the original problem (HINT use variables x_3', x_4', x_5', z'). [5]

(ii) How large can p be before the original solution ceases to be optimal? [3]

(iii) Suppose that $p = 3$. Modify the original simplex tableau to take account of the new problem. [5]

(iv) Determine the entering and departing variables for this new tableau and give your reasoning. [2]

(v) Find the new optimal value of z, assuming that it will be obtained from a single iteration of the Simplex method on the new tableau. [You do NOT have to find the complete optimal tableau.] [2]

Question 4

Four warehouses are supplied by three factories. The supply available from each factory, the demand at each warehouse and the cost per unit of transporting goods from the factories to the warehouses are summarised in the following table:

	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	Supply
Factory 1	5	4	7	5	8
Factory 2	6	5	8	8	12
Factory 3	7	5	6	7	10
Demand	7	5	6	12	

(i) Use the Least-Cost method to find an initial basic feasible solution of this problem. [**Do NOT use the North-West Corner method.**] [4]

(ii) Find the optimal solution of this problem, i.e. the solution that minimises the transportation costs, **clearly showing and explaining your working at each step**. [12]

(iii) What is the minimum total transportation cost? [2]

(iv) What is the effect on the optimal solution and the total transportation cost of an increase in the cost of transporttatioj from Factory 3 to Warehouse 2 from 7 to 9? [2]

(v) What is the effect on the optimal solution and the total transportation cost of a decrease in the cost of transporttatioj from Factory 3 to Warehouse 2 from 7 to 3? [5]

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