

UNIVERSITY OF SURREY[©]

B. Sc. Undergraduate Programmes in Computing

Level HE1 Examination

Module MAT1014 Quantitative Methods

Time allowed – 2 hrs

Spring 2009

Attempt **ALL** questions in **Section A** , this section is worth 40% of your overall mark.

Answer **THREE** questions from **Section B**, this section is worth 60% of your overall mark.

There is a formula sheet which accompanies this examination.

SEE NEXT PAGE

Section A

Answer the questions in this section using the answer sheet provided.

Question 1

What is the range of the function $y = 4x^2 + 1$ when $-3 \leq x < 1$? [2]

Question 2

If $f(x) = x^2 + 1$ and $g(x) = 1 - x$, what is $f(g(x))$? [2]

Question 3

Is the function $f(x) = x^2 \cos x$ odd, even or neither? [1]

Question 4

Is the function $f(x) = x^3 - x^6$ odd, even or neither? [1]

Question 5

What is the inverse of the function $f(x) = 1 - 3x$? [2]

Question 6

What is the smallest positive value of c for which $3 \cos x - c \leq -1$ for all x ? [2]

Question 7

The graph of the function $f(x) = \cos(3x) + x - 1$ crosses the x -axis at $x = 0$. Where does the graph of the function $g(x) = \cos(3x - 6) + x - 3$ cross the x -axis? [2]

Question 8

What is the period of the function $y = 2 \cos(3x + 1)$? [1]

Question 9

If $e^{2 \ln x^3} = x^a$, what is the value of a ? [2]

Question 10

What is 225 degrees in radians? [1]

Question 11

What is $\frac{3\pi}{4}$ radians in degrees? [1]

Question 12

A right-angled triangle has hypotenuse of length 10 and an angle of $\frac{\pi}{4}$. What are the lengths of the two other sides of the triangle? [2]

Question 13

A right-angled triangle has two shorter sides of length 21 and 28. What is the length of the hypotenuse? [2]

Question 14

A cake has radius 21cm and is cut into 6 equal slices. What is the area of one slice (in cm^2)? [2]

SEE NEXT PAGE

Question 15

Find the equation of the straight line which passes through the two points $(1, 4)$ and $(5, 0)$. [2]

Question 16

Find the centre and radius of the circle given by

$$x^2 + y^2 - 6x + 2y + 4 = 0. \quad [3]$$

Question 17

Find all the solutions of the equation

$$4x^2 - 3x - 2 = 0. \quad [2]$$

Question 18

The supply Q_S and the demand Q_D for a product are related to its price P by the following equations:

$$\begin{aligned} Q_D &= 4 + 5P \\ Q_S &= 6 - 2P \end{aligned}$$

Find the equilibrium price P_E and the equilibrium quantity Q_E . [2]

Question 19

Expand fully $(4x - 3y)^3$. [2]

Question 20

Find the terms up to order x^2 in the expansion of $\left(1 - \frac{x}{3}\right)^{11}$. [2]

Question 21

Find the derivative $\frac{dy}{dx}$ in each of the following cases:

$$(a) y = 6x^2 - x^4, (b) y = -\frac{1}{x^3}, (c) y = e^{\frac{x}{2}} + \cos 2x. \quad [3]$$

Section B

Answer the questions in this section in the **answer book** provided.

Question 1

(a) Find the derivative of the following functions in its simplest form:

$$(i) y = x^2 \sin 3x, \quad (ii) y = \frac{e^x}{\cos x}, \quad (iii) y = \ln \left(\frac{x+2}{x-2} \right). \quad [9]$$

(b) Find the x and y co-ordinates of and characterise all the local maxima and minima of $y = x^3 - 6x^2 + 9x + 3$. [11]

Question 2

(a) Evaluate the following integrals:

$$(i) \int 2x^3 - 3x^2 + 7 \, dx, \quad (ii) \int_0^{\frac{\pi}{6}} \sin 3x \, dx, \quad (iii) \int_0^1 (e^x + e^{-x})^2 \, dx. \quad [10]$$

(b) Show that the equation $x^3 + 2x - 1 = 0$ has a root in the interval $[0, 0.5]$. Use the Newton Raphson Method to find this root accurate to 2 d.p. [10]

Question 3

(a) A sum of £20,000 is invested in each of three accounts that pay respectively

- (i) 6% per annum simple interest;
- (ii) 6% per annum compound interest, compounded annually;
- (iii) 6% per annum compound interest, compounded monthly.

How much is in each account after 24 years?

(iv) How much is required to be invested initially in an account paying simple interest at 6% per annum in order to give a return after 15 years which is the same as if £20,000 were invested in an account paying 6% per annum compound interest, compounded continuously, for the same period? [12]

(b) A sum of £60 per month is paid into a savings account which offers a return of 5% per annum, compounded monthly.

- (i) How much money will be in the account after 12 years?
- (ii) What sum should be invested each month in order to accumulate a total of £15,000 in the account after 12 years? [8]

SEE NEXT PAGE

Question 4

- (a) Solve the following system of linear equations using Gaussian elimination:

$$\begin{aligned}x + 2y + 3z &= 14 \\ -2x + y + z &= 3 \\ x - y + 2z &= 5\end{aligned}$$

[11]

- (b) Determine the rank of the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 3 \\ 2 & 3 & 4 & 9 \\ 3 & 4 & 5 & 12 \end{pmatrix}.$$

Hence, or otherwise, determine the number of solutions of the linear system of equations

$$\begin{aligned}x + y + z &= 3 \\ 2x + 3y + 4z &= 9 \\ 3x + 4y + 5z &= 12\end{aligned}$$

[9]

Question 5

- (a) Solve the following differential equations:

(i) $\frac{dy}{dx} = 2 \sin 3x$, given that $y(0) = 3$;

(ii) $\frac{d^2y}{dx^2} = 25y$, given that $y(0) = 4$ and $\left. \frac{dy}{dx} \right|_{x=0} = 0$.

[9]

- (b) Consider the following model where supply (
- Q_S
-) and demand (
- Q_D
-) are related to the price (
- P
-) of a product by

$$\begin{aligned}Q_D &= 9 - 2P \\ Q_S &= 4 + 3P\end{aligned}$$

- (i) If the rate of change of price is equal to 3 times the excess demand, write down the differential equation that the price P must satisfy.
- (ii) Solve this differential equation for P as a function of time t , using the additional condition that the price has the value 8 when $t = 0$.

[11]