

CM 1021 Mathematical Methods for Computing I

Exercise Sheet 1

1. If $f(x) = x^2 + 2x$ and $g(x) = x - 1$, find $f(g(x))$ and $g(f(x))$.
2. If $f(x) = 3x + 2$ and $g(x) = 4x + c$, find $f(g(x))$ and $g(f(x))$ and find a value of c for which $f(g(x)) = g(f(x))$.
3. Find the range of the functions:
 - (a) $y = x^2 + 3, -4 \leq x \leq 1$
 - (b) $y = 4 - 3x^2, -2 \leq x \leq 3$.
4. Find the inverse of the function $f(x) = 4x - 7$. Verify that $f(f^{-1}(x)) = x$ and that $f^{-1}(f(x)) = x$.
5. Find the inverse of the function $f(x) = \frac{2x - 3}{x - 4}, x \neq 4$.
6. Determine whether the following functions are odd, even or neither.
 - (i) $x^4 + 3x^2 - 2$, (ii) $x^3 - x^2$, (iii) $x^3 - 3x$
7. Sketch, on the same axes, the graphs of
 - (a) x^2 and $x^2 - 4$ for $-4 \leq x \leq 4$.
 - (b) x^2 and $(x + 1)^2$ for $-4 \leq x \leq 4$.
8. Sketch the graph of (i) $3x - 5y - 30 = 0$, (ii) $5x + 3y - 15 = 0$ (iii) $y = |x - 1|$
9. Sketch the graph of (i) $y = 3e^x$, (ii) $y = x^2 - 3x + 2$
10. Find the equation of the straight line which passes through the points $(1, -2)$ and $(-2, 7)$. Find the points at which the line crosses the x -axis and the y -axis.
11. Find the centre and radius of the circle given by
$$x^2 + y^2 - 6x + 2y - 15 = 0.$$
12. Use the rules of logarithms to simplify the following:
 - (a) $\ln x + \ln x^2$, (b) $\ln e^3$, (c) $\ln(\frac{1}{2}) + \ln 4$,
 - (d) $e^{5 \ln x}$, (e) $\ln 6 - \ln 3$, (f) $\ln(x + xy) - \ln x$.

More Challenging Questions

13. Find the domain and the range of the functions:

$$(a) f(x) = \sqrt{1-x^2} \quad (b) g(x) = \sqrt{x^2+x-2} \quad (c) h(x) = \frac{1}{x^2+x-2}.$$

14. Show that if $f(x)$ and $g(x)$ are both odd functions, then $h(x) = f(x) \times g(x)$ is an even function.

15. Show that $f(x) = \frac{x+1}{x-1}$ is one-to-one. Find f^{-1} and show that your solution is correct. Can you explain the form of f^{-1} ?

16. Evaluate

$$(a) \lim_{x \rightarrow \infty} \frac{2x+3}{x-1} \quad (b) \lim_{x \rightarrow 3} \frac{x-3}{x^2-9}$$

17. Sketch the following graphs

$$(a) y = \frac{x^2+5x+4}{4x+1}, \quad (b) y = \frac{4x+1}{x^2+5x+4}$$