

**C0M 1021 Mathematical Methods for Computing I**  
**Exercise Sheet 4**

1. Let  $\mathbf{u} = (-3, 2, 1, 0)$ ,  $\mathbf{v} = (4, 7, -3, 2)$ ,  $\mathbf{w} = (5, -2, 8, 1)$ ,  $\mathbf{x} = (1, 2, 3, 4)$

(a) Find (i)  $\mathbf{v} - \mathbf{w}$  (ii)  $3\mathbf{w} + 2\mathbf{x} - \mathbf{u}$  (iii)  $(6\mathbf{w} + 2\mathbf{v}) - (4\mathbf{x} - \mathbf{u})$

(b) Find (i)  $\|\mathbf{u}\|$  (ii)  $\|\mathbf{w} + \mathbf{x}\|$  (iii)  $\|\mathbf{x} - \mathbf{u}\|$

(c) Find (i)  $\mathbf{u} \cdot \mathbf{v}$  (ii)  $\mathbf{w} \cdot \mathbf{x}$

(d) Find a vector  $\mathbf{y}$  such that  $5\mathbf{y} - 2\mathbf{v} = 2(\mathbf{w} - 5\mathbf{x})$

(e) Find the value of all scalars  $k$  such that  $\|k\mathbf{u}\| = 14$

2. One student bought 2 CD's and 3 DVD's for £65 while another student bought 4 CD's and 1 DVD for £55. How much does a CD and a DVD cost?

3. Solve the following systems of linear equations:

(a)  $x + 3y = -1$

$$4x - 2y = 10$$

(b)  $2x - 5y = -8$

$$4x + y = 6$$

(c)  $x + 2y + z = -3$

$$x + 3y + 3z = -9$$

$$-x + 3y - 2z = 17$$

(d)  $x + 2y + 5z = 5$

$$2x + y + 3z = 8$$

$$3x + 2y + z = 10$$

4. For the two matrices

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & -1 & 4 \\ -2 & 5 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 0 & 3 \\ -1 & -2 & 3 \end{bmatrix},$$

find (i)  $B - 3A$ , (ii)  $AB$  and (iii)  $BA$ .

5. For the matrix

$$A = \begin{bmatrix} 1 & 3 & 1 \\ -2 & 5 & 3 \\ -3 & 0 & 1 \end{bmatrix},$$

show that the matrix

$$A^{-1} = \begin{bmatrix} -5 & 3 & -4 \\ 7 & -4 & 5 \\ -15 & 9 & -11 \end{bmatrix}$$

is the inverse of  $A$  and hence find the solution of the following system of linear equations:

$$\begin{bmatrix} 1 & 3 & 1 \\ -2 & 5 & 3 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 1 \end{bmatrix}.$$

6. Show that the following linear system of equations does not have a solution: [Hint: Use Gaussian elimination]

$$\begin{aligned}x + 2y + z &= 4 \\2x + 5y - 2z &= 6 \\x + 4y - 7z &= 6\end{aligned}$$

7. Solve the following system of equations

$$\begin{aligned}2u + v - 2w + 3x &= 2 \\v + 3w - 3x &= 1 \\2u + 3v + 2w - x &= 0 \\-4u - 3v + 5w - 4x &= 0\end{aligned}$$

8. Find the inverse of the matrix

$$\begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix},$$

and hence solve the system

$$3x + 4y = 13, \quad -2x + 5y = -1.$$

9. Find the eigenvalues and eigenvectors of the following matrices

$$(i) \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}, (ii) \begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}, (iii) \begin{bmatrix} 4 & 1 \\ 6 & 3 \end{bmatrix}$$

### More Challenging Questions

10. Find two non-zero  $2 \times 2$  matrices  $A$  and  $B$  such that their product is the zero matrix
11. Is  $(A + B)(A - B) = A^2 - B^2$  for matrices  $A$  and  $B$ ? if it is not true in general can you find a pair of matrices for which the result is true.

12. Let

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},$$

Find  $A^2$ ,  $A^3$  and  $A^4$ . What is  $A^n$  for positive integer  $n$ ?

13. Find the eigenvalues and eigenvectors of the following matrices

$$(i) \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}, (ii) \begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{bmatrix}$$