

CM 1021 Mathematical Methods for Computing I

Exercise Sheet 7

1. Find the value of

$$(a) \sum_{r=1}^{23} r \quad (b) \sum_{r=7}^{13} (3r - 2)$$

2. Find the sum of all even positive integers less than 2000.

3. The boring of a well costs £5 for the first metre depth, £11 for the second and £17 for the third. The costs for each successive metre continue in the same arithmetic series. Find the cost of boring a well of (a) 50 metres and (b) 100 metres

4. Find the 10th term and the sum of the first ten terms of the following geometric series:

$$a + ar + ar^2 + ar^3 \dots$$

when

$$(i) a = 10, r = 2 \quad (ii) a = 10, r = -2 \quad (iii) a = 10, r = -\frac{1}{2}$$

$$(iv) a = 3, r = 6 \quad (v) a = 3, r = -6$$

5. Find the sum to n terms and the sum to infinity (where the series converges) of the following series:

$$(i) \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \dots \quad (ii) 16 - 8 + 4 - \dots \quad (iii) 8 - 12 + 18 - \dots$$

6. Find the first 3 terms in the expansion of the following in **ascending** powers of x

$$(a) (1 - x)^{23} \quad (b) (1 + x)^{15}$$

7. Find the first three terms in **descending** powers of x of $(5x - 3)^7$.

8. Expand $(1 + x)^{10}$, in ascending powers of x obtaining the first four terms. Hence obtain an approximation of 0.998^{10}

9. Expand the following in series in ascending powers of x up to and including the term in x^3 , simplifying as much as possible and stating the range of values of x for which the series is valid:

$$(a) \frac{1}{(1 + x)^2} \quad (b) \frac{1}{(1 - x)^3} \quad (c) \frac{2}{(1 - \frac{x}{2})^2} \quad (d) (4 + 3x)^{\frac{1}{2}}$$

$$(e) (1 - x)^{\frac{3}{4}} \quad (f) \frac{1}{(100 + x)^{\frac{1}{2}}} \quad (g) (1 - 3x)^{\frac{1}{3}}.$$

More Challenging Questions

10. Find the sum of all the integers between 0 and 200 that are not divisible by 4.

11. The first and third term of an arithmetic series are $\frac{2}{3}$ and $\frac{3}{2}$ respectively. Find the common difference and the sum of the first twelve terms.

12. Find the value of

$$(a) \sum_{n=4}^9 (1.5)^n \quad (b) \sum_{n=2}^{10} 4 \left(\frac{3}{4}\right)^{n-1}$$

13. Suppose that the price of a house increases at a constant 3% per annum. At the start of 1994 it was worth £45,000. What was the value at the start of 1980? What would the value be at the start of 2020? 2200?

14. The coefficients of x , x^2 and x^3 in the expansion of $(1+x)^n$ form the first three terms of an arithmetic series. Show that $n = 7$.

15. Find the coefficient of x^{12} in the expansion of $(x+y)^{18}$. Evaluate this term with $x = 2, y = \frac{1}{3}$.

16. Given that $|x| < 1$, expand $\left(\frac{1+x}{1-x}\right)^{\frac{1}{3}}$ up to and including the term in x^2

17. The coefficients of x and x^2 in the expansion of $(1+px+qx^2)^{-2}$ in ascending powers of x are 4 and 14. Find p and q