CM 1021 Mathematical Methods for Computing I

Supplementary Exercise Sheet A

1. Find the domains of the following functions:

   (a) (i) $y = \sqrt{x-3}$ (ii) $y = \sqrt{1-2x^2}$ (iii) $y = \dfrac{1}{1-x}$

   (b) (i) $y = \dfrac{1}{1-x^2}$ (ii) $y = \dfrac{1}{\sqrt{x^2-3}}$ (iii) $y = \dfrac{1}{\sqrt{x^2+3x+1}}$

   (c) (i) $y = \dfrac{1}{\sqrt{x^2-4x+3}}$ (ii) $y = \ln(x-4)$ (iii) $y = \ln(3-x^2)$.

2. Find the range of the following functions, where $x \in \mathbb{R}$ unless otherwise specified:

   (a) (i) $y = x^2 + 2x + 5$ (ii) $y = x^2 + 7x + 13$ (iii) $y = |3x - 1|$

   (b) (i) $y = e^{3x-1}$ (ii) $y = 3 + 2x - x^2$ (iii) $y = 1 - e^{-x}, \quad x \geq 0$

   (c) (i) $y = 2^x, \quad x \leq 5.$

3. The functions $f(x), g(x)$ and $h(x)$ are defined as follows:

$$f(x) = 4 + 3x \qquad g(x) - = 7 - 2x \qquad h(x) = 1 + x^2$$

   Find the following

   (a) (i) $f^{-1}(x)$ (ii) $g^{-1}(x)$ (iii) $f(g(x))$

   (b) (i) $h(g(x))$ (ii) $f^{-1}(g^{-1}(x))$ (iii) $h(h(x))$

   (c) (i) $h(h(h(x)))$ (ii) $f(f(x))$ (iii) $f(f(f(x)))$ (iv) $f^{[n]}(x)$

4. Sketch the following graphs, showing clearly the values of any points of intersection with the x and y axes as well as any local maxima and minima.

   (a) $3x + 2y - 6 = 0$

   (b) $y = 8 - x$

   (c) $y = x^2 + 6x + 5$

   (d) $y = x^2 + 6x + 10$

   (e) $y = 5 - 6x - x^2$

   (f) $y = \dfrac{3\,x^2 - 5}{x^2 - 4\,x + 3}.$

5. Find the equations of the straight lines between each of the following pairs of points:

   (a) (i) $(9,3), \quad (-1,8)$ (ii) $(4,3), \quad (-12,19)$ (iii) $(-1,-3), \quad (-5,1)$.

6. Find the coordinates of the centre and the radius of each of the following circles:

   (a) $x^2 - 4x + y^2 - 4y - 28 = 0$

   (b) $x^2 + 6x + y^2 + 10y - 25 = 0$

   (c) $9x^2 + 12x + y^2 - 10y - 20 = 0$

   (d) $16x^2 - 24x + 9y^2 - 6y - 15 = 0$.

7. Find the inverses of the following functions and in each case sketch the graph of the function and its inverse on the same plot:

   (a) (i) $f(x) = \dfrac{2-3x}{5x+6}$ (ii) $g(x) = \dfrac{3x-7}{1-2x}$.

8. Find the inverses of the following functions:

   (a) (i) $h(x) = e^{2x-3}$ (ii) $p(x) = \ln(2-3x)$ (iii) $q(x) = 10^{3x}$

9. Calculate:

   (a) (i) $4^5 \div 4^{-3}$ (ii) $5^4 - 4^5$ (iii) 3^{2^3} (iv) $\left(3^2\right)^3$.

10. Simplify as far as possible:

    (a) (i) $\dfrac{x^2y^3z^4}{x^3y^2z^5}$ (ii) $\dfrac{x^{\frac{1}{2}}y^{\frac{3}{2}}x^{\frac{1}{3}}}{xy^{\frac{1}{2}}z^2}$ (iii) $\ln 64$

    (b) (i) $\dfrac{1}{2}\ln 3 - \ln 9 + 3\ln 3$ (ii) $\ln x^6 - 4\ln x + 3\ln 3$

11. State whether (or where) the following functions are one to one, odd or even or neither, increasing or decreasing or periodic (state the period):

    (a) (i) x^3 (ii) $x^2 + 4$ (iii) $x^4 + 3x^2 + 2$ (iv) $x^3 - x^5$ (v) $\cos\dfrac{2x}{3}$

    (b) (i) $\tan\dfrac{2x}{3}$ (ii) $\sin(4x-1)$ (iii) $x^2\sin x$ (iv) $x\sin 3x$ (v) e^{-3x}

    (c) (i) $4 - e^{-3x}$ (ii) $e^x - \ln x$.

12. Find the limit of each of the following expressions as $n \to \infty$:

    (a) (i) $\dfrac{2n^3-1}{n^3+64n^2}$ (ii) $\dfrac{1-n}{1+n}$ (iii) $\dfrac{1-3n^2}{1-n^3}$ (iv) $1 + 3e^{-n}$