Chapter 6 Exercises

Solutions

1. *A company currently replenishes its stock by ordering enough supply to cover a 1 month demand. The annual demand of the item is 1500 units. It costs £20 each time an order is placed. The holding cost per unit inventory per month is £2 and no shortages are allowed.*

   (a) *Determine the optimal order quantity and the time between orders.*

   (b) *Determine the difference in annual inventory costs between the optimal policy and the current policy of ordering a 1 month supply 12 times a year.*

   With the usual symbols for the parameters and variables, $\beta = 1500/12 = 125$. $K = 20, h = 2$.

   (a) $Q_0^* = \sqrt{2K\beta/h} = 50$. $t^* = Q_0^*/\beta = 0.4$ months.

   (b) Optimal policy: Monthly cost $= T(Q_0^*) = \sqrt{2K\beta h} = 100$.

   Optimal yearly cost = £1200.

   Current policy: $Q_0 = 125$ (= monthly demand).

   Monthly cost $= T(125) = K\beta/125 + 125h/2 = 145$. Yearly cost = £1740. Difference between annual costs = £540.

2. *An item is consumed at the rate of 30 units per day. The holding cost per unit per day is £0.05 and the setup cost is £100. Suppose that no shortage is allowed and the purchasing cost per unit is £10 for any quantity less than $q = 300$, and is £8 otherwise. Find the economic lot size. What is the answer if $q = 500$?*

   In the price-break model, $\beta = 30$, $h = 0.05$, $K = 100$, $q = 300$, $c_1 = 10$, $c_2 = 8$. The two cost per unit time functions are then

$$\begin{aligned} T_1(Q_0) &= 300 + 3000/Q_0 + Q_0/40 \\ T_2(Q_0) &= 240 + 3000/Q_0 + Q_0/40 \end{aligned}$$

   The minimum of both of these occurs at $Q_m = \sqrt{2K\beta/h} = 346.41$. Since $q < Q_m$, then q is in Zone I and so $Q_0^* = Q_m = 346.41$.

   If $q = 500$ then we must find q_1 by solving $T_1(Q_m) = T_2(q_1)$. Substituting the known values gives the equation

$$77.32 = 3000/q_1 + 0.025q_1.$$

   The solution of this equation which is greater than Q_m is $q_1 = 3053.5$. Thus, $Q_m < q < q_1$ and so q is in Zone II. The optimal order quantity is therefore $Q_0^* = q = 500$ units.

3. *An item sells for £4 per unit but a 10% reduction is offered for lots of size 150 or more. A company that consumes this item at the rate of 20 items per day wants to decide whether to take advantage of the discount. The setup cost for ordering is £50 and the holding cost per unit per day is £0.30. Should the company take advantage of the discount?*

   In the price-break model, $\beta = 20$, $h = 0.3$, $K = 50$, $q = 150$, $c_1 = 4$, $c_2 = 3.6$. Thus, $Q_m = \sqrt{2K\beta/h} = 81.64$ and so $q > Q_m$. Thus, we must find q_1 by solving $T_1(Q_m) = T_2(q_1)$. Substituting the known values gives the equation

$$32.495 = 1000/q_1 + 0.15q_1.$$

   The solution of this equation which is greater than Q_m is $q_1 = 179.49$. Thus, q is in Zone II and so $Q_0^* = 150$ units. The company should take the discount.

4. *Demand for a product is 600 units per month. The set up cost for placing an order is £25, the unit cost of each item is £3 and the holding cost is £0.05 per month.*

   (a) *Assuming that shortages are not allowed determine how often to order and what the optimal order quantity should be.*

   Here $\beta = 600, K = 25, h = 0.05$, thus

$$Q_0^\star = \sqrt{\frac{2K\beta}{h}} = \sqrt{\frac{2 \times 25 \times 600}{0,05}} = 775 \quad \text{to the nearest whole number}$$

   The frequency of ordering is $\frac{Q_0^\star}{\beta} = 1.29$ months or every 5.6 weeks

   (b) *Assuming that shortages <u>are</u> allowed but there is a shortage cost of £0.2 per week, determine how often to order and what the optimal order quantity should be.*

   We have $p = 0.02$. hence

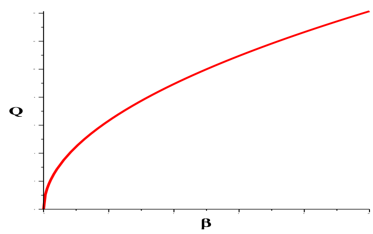
$$Q_0^\star = \sqrt{\frac{2K\beta}{h}} \times \sqrt{\frac{p+h}{h}} = \sqrt{60000} \times \sqrt{1.25} = 886 \quad \text{to the nearest whole number}$$

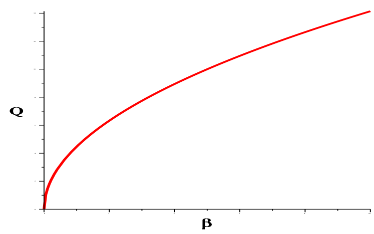
   The frequency of ordering is $\frac{Q_0^\star}{\beta} = 1.44$ months or every 6.3 weeks

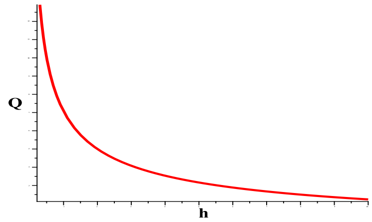
5. We know that

$$Q_0^\star = \sqrt{\frac{2K\beta}{h}}$$

   - *Sketch graphs to show how $Q_0^\star$ changes as each of K, β and h change separately.*







- *What happens to the value of $Q_0^\star$ as a result of the following (separate) changes*

(a) *Set up costs reduced to 25% of its original value.*

(b) *Demand rate becomes 4 times as large.*

(c) *The unit holding cost is reduced to 25% of the original value*

$$Q_0^\star = \sqrt{\frac{2K\beta}{h}}$$

If K reduced to 25% the original value then $Q_0^\star$ will halve.
If β becomes 4 times as large then $Q_0^\star$ will double.
If h is reduced to 25% the original value then $Q_0^\star$ is increased by a factor of $\sqrt{\frac{1}{\frac{1}{2}}} = 2$

6. *What is the effect on $Q_0^\star$ as a result of*

(a) *both changes in (a) and (b) above occuring simultaneously*
No change

(b) *both changes in (a) and (c) above occuring simultaneously*
$Q_0^\star$ increases by a factor of 4.