CM 1021 Mathematical Methods for Computing I

Exercise Sheet 2 - solutions

1. Convert the following angles

   (a) from radians to degrees: $\pi/4$, 3π, $-5\pi/2$, 1.36, 2.45

   $$45^o, \quad 270^o, \quad -450^o, \quad 77.9222^o, \quad 140.3746^o$$

   (b) from degrees to radians: 90^o, 135^o, 315^o

   $$\frac{\pi}{2}, \quad \frac{3\pi}{4}, \quad \frac{7\pi}{4}, \text{ or } 1.5707, \quad 2.3561, \quad 5.4977$$

2. A pizza has radius 20cm. A slice is cut from the pizza with an angle of 48^o at the point. What is the area of the slice?

   $$\text{Area} = \frac{48}{360} \times \pi \times 20^2 = \frac{160\pi}{3}$$

3. Calculate angles A and B, and side a in the left-hand figure, and angle A and sides b and c in the right-hand figure. Both are right-angled triangles.

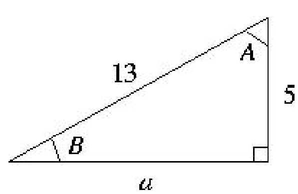
   (i) $a = \sqrt{13^2 - 5^2} = 12$

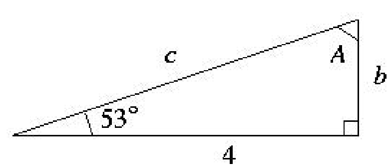
   $$A = \cos^{-1}\left(\frac{5}{13}\right) = 1.1760^c = 67.3801^o$$

   $$B = \sin^{-1}\left(\frac{5}{13}\right) = 0.3948^c = 22.6199^o$$

   (ii) $A = (90 - 53)^o = 37^o$, $\quad c = \dfrac{4}{\cos 53^o} = 6.6466$

   $b = \sqrt{c^2 - 16} = 5.3082$





4. Expand (a) $\sin(x + \pi/6)$ (b) $\tan(x + \pi/4)$ (c) $\cos(x - \pi/3)$

   (a) $\frac{1}{2}(\sin(x)\sqrt{3} + \cos(x))$ (b) $\dfrac{1 + \tan x}{1 - \tan x}$ (c) $\frac{1}{2}(\sin(x)\sqrt{3} + \cos(x))$

5. Without using a calculator find the exact value of
   (a) $\cos(15^o)$ (b) $\sin\left(22\frac{1}{2}^o\right)$ (c) $\tan(75^o)$

   (a) $\sqrt{\frac{1}{4}(2 + \sqrt{3})}$ (b) $\sqrt{\frac{1}{2}(1 - \frac{1}{\sqrt{2}})}$ (c) $\dfrac{\sqrt{3} + 1}{\sqrt{3} - 1}$

6. Solve the following triangles using the sine or cosine rules as appropriate, giving your answers to 2 d.p.s and expressing angles in degrees

   (a) $a = 7, b = 6, c = 9$
   $A = 50.98^o, B = 41.75^o, C = 87.27^o$

   (b) $A = 25^o, b = 27, c = 32$
   $a = 13.67, B = 56.58^o, C = 98.42^o$

   (c) $A = 74^o, B = 10^o, c = 14$
   $C = 94^o, a = 13, 53, b = 2.44$

7. Find the period of (a) $\cos 5x$, (b) $\sin 6x$, (c) $\cos(3x - 2)$

   Using the formula, period $= \frac{2\pi}{\omega}$ the solutions are (a) $\frac{2\pi}{5}$, (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$

8. What is the distance between the points whose coordinates are $(4, 11, -5)$ and $(7, -3, -1)$

   Distance $=\sqrt{(4-7)^2 + (11+3)^2 + (-5+1)^2} = \sqrt{221}$

More Challenging Questions:

9. Find $\sin 3x$ in terms of $\sin x$

$$\begin{aligned}\sin 3x = \sin(x + 2x) &= \sin x \cos 2x + \cos x \sin 2x \\ &= \sin x(1 - 2\sin^2 x) + \cos x.2 \sin x \cos x \\ &= \sin x - 2\sin^3 x + 2 \sin x \cos^2 x \\ &= \sin x - 2\sin^3 x + 2\sin x(1 - \sin^2 x) \\ &= 3 \sin x - 4\sin^3 x\end{aligned}$$

10. If $\tan \frac{x}{2} = t$, find expressions for $\sin x, \cos x$ and $\tan x$ in terms of t.

    In the triangle with angle $\frac{x}{2}$ the sides are $1, t, \sqrt{1+t^2}$.
    Thus $\sin \frac{x}{2} = \frac{t}{\sqrt{1+t^2}}$ and $\cos \frac{x}{2} = \frac{1}{\sqrt{1+t^2}}$.
    Now use $\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$ to get $\sin x = \frac{2t}{1+t^2}$.
    Then $\cos x = \sqrt{1 - \frac{4t^2}{(1+t^2)^2}} = \frac{1-t^2}{1+t^2}$.
    Finally $\tan x = \frac{2t}{1-t^2}$ from the double angle formula.