

## CM 1021 Mathematical Methods for Computing I

### Exercise Sheet 3

1. Solve the following pairs of simultaneous equations

(a)  $3x + y = 7, 2x - 3y = 1$

(b)  $5x + 10y = 18, 2x - 4y = 6$

(c)  $2x + 3y = 16, x - y = 10$

2. Make  $x$  the subject of the following equations

(a)  $y = \frac{3x + 4}{4x + 3}$

(b)  $y = \frac{x^2 - 4}{x^2 + 4}$

3. Solve the following quadratic equations

(a)  $x^2 - 11x + 10 = 0$

(b)  $x^2 + x - 3 = 0$

(c)  $5x - 6x^2 - 1 = 0$

(d)  $x^2 - 3x - 1 = 0$

(e)  $x^4 + 3x^2 - 4 = 0$

4. Solve the following inequalities

(a)  $x - 3 \leq 4 - \frac{x}{2}$

(b)  $3x + 1 < -2 - x$

(c)  $x^2 - 16 < 0$

(d)  $x^2 + 4x + 3 > 0$  [Hint: factorise the left hand side and sketch a graph]

5. Sketch the region of the  $x, y$  plane corresponding to (a)  $y > x + 2$  (b)  $x + y \leq 5$  and

(c)  $y \leq x^2$ .

6. For what range of values of  $c$  does the equation  $3x^2 - 4x - c = 0$  have at least one real root?

### More Challenging Questions:

7. Make  $x$  the subject of the following formulae

$$(a) \quad y = \frac{2x^2 + 1}{4x^2 + 3}$$

$$(b) \quad y = \frac{2x^2 + 1}{4x + 3}$$

$$(c) \quad y = \frac{e^{x^2} + 1}{e^{x^2} - 1}$$

8. Solve the following equations

$$(a) \quad e^x + 21e^{-x} - 10 = 0$$

$$(b) \quad 3^{2x} - 5 \cdot 3^{x+1} + 36 = 0$$

9. Sketch the area on the graph represented by  $x^2 + y^2 < 9$

10. The hyperbolic cosine (cosh) and the hyperbolic sine (sinh – pronounced “shine”) are defined by

$$\cosh x = \frac{1}{2}(e^x + e^{-x}), \quad \sinh x = \frac{1}{2}(e^x - e^{-x}).$$

(a) Are  $\cosh x$  and  $\sinh x$  even, odd or neither?

(b) Simplify the following expressions as far as possible.

$$(a) \quad 2 \cosh(\ln x), \quad (b) \quad \cosh 5x + \sinh 5x, \quad (c) \quad \sinh(2 \ln x).$$

(c) Solve  $\cosh x = 2$

11. Solve  $x^3 - 6x^2 + 11x - 6 = 0$  [Hint: look for a factor]