Operations Research for Computing, Spring 2010, Coursework 1

Please hand this is to the undergraduate office no later than 9:00 a.m. on Monday March 22 2010, or if submitted electronically, to the lecturer, not later than 2359 on Sunday March 21 2010. Remember that this coursework must be ***all your own work****.*

1. Angela knits teddy bears and rag dolls that she sells to raise money for charity. They are very popular and she always sells them all. Each week she has 12 balls of wool, 10 bags of stuffing and 20 hours available for her knitting. Each teddy bear requires 2 balls of wool and 2 bags of stuffing and takes 2 hours to make, while each rag doll uses 2 balls of wool, one bag of stuffing and takes 4 hours to make. The profit on a teddy bear is £2 and on a rag doll is £3.

   Angela wants to know how many of each type of toy she should make each week in order to raise the most money for her charity.

   (a) What are the decision variables for this problem? 2

   (b) What is the objective function? 2

   (c) State the constraints as inequalities. 5

   (d) Sketch the feasible region, indicating the coordinates of each of the corner points. On your diagram, show the contour associated with a weekly profit of £13. 5

   (e) Using this contour or otherwise (but NOT by the Simplex Method), determine the optimum solution to the problem, justifying your approach. . 4

   (f) How many toys of each kind should Angela make each week? 2

   (g) Angela has noticed that the demand for teddy bears has increased a lot recently, so she is considering putting up their price. How high does the profit on a teddy bear have to be before the optimal solution in part (e) changes?

   How many toys of each kind should Angela make each week when the profit on a teddy bear exceeds this amount? 5

2. (a) Put the following problem in standard form. Do NOT attempt to solve it.

   Minimize

   $$z = 6x_1 - x_2 - 3x_3$$

   subject to the constraints

   $$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ x_1 + 2x_2 - x_3 &\leq 2 \\ 2x_1 - 3x_2 - 2x_3 &\geq -3 \end{aligned}$$

   and

   $$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \quad \text{no restriction}$$

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(b) Consider the following problem:

Maximize

$$z = 2x_1 + x_2 - x_3$$

subject to the constraints

$$x_1 + 3x_2 + x_3 \leq 21$$
$$6x_1 + x_2 + 2x_3 \leq 24$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

i. Put this problem in standard form. 2

ii. Find an initial basic feasible solution, stating which are the basic variables and which are the non-basic variables. 2

iii. Construct the initial tableau for the Simplex Method. 3

iv. Use the tableau form of the Simplex Method to find the optimal solution of the problem. At each stage, state the entering and departing variables from the basis. From the optimal tableau, determine the optimal value of z and state the values of all the variables at this solution. 11

3. A farmer grows and sells barley, oats and wheat for profit. However, there are constraints on the available land, finance and labour, which are measured in acres, £sterling and person-hours respectively.

This problem involves three decision variables x_1, x_2 and x_3 which are the areas (measured in acres) devoted to growing barley, oats and wheat respectively. There are also three slack variables x_4, x_5 and x_6 associated with the constraints on land, finance and labour respectively.

The profit made on barley, oats and wheat is £15 per acre, £35 per acre and £25 per acre respectively.

The optimal tableau for this problem is as follows:

Basic	z	x_1	x_2	x_3	x_4	x_5	x_6	Solution
x_1	0	1	-1	0	$\frac{3}{2}$	0	$-\frac{1}{2}$	2000
x_3	0	0	2	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1000
x_5	0	0	-3	0	0	1	-1	1000
z	1	0	0	0	10	0	5	55,000

(a) i. What is meant by the *shadow price* of a resource?
What is the significance of a zero and a non-zero shadow price? 3

ii. What is the shadow price associated with each of the resources in the above problem? . 2

iii. Considering only the shadow prices, which resource would you alter to increase the maximum profit? Justify your answer. 2

iv. Land costs £10,000 per acre and labour costs £4.25 per person-hour. Bearing this in mind, would you recommend that the farmer buys more land, hires more labour or does neither? If your recommndation is positive, calculate how much addtional resource should be obtained while maintaining the same optimal solution (corner point). 7

(b) i. The farmer wants to maximise his profit. State the objective function which is to be maximised for this problem. 2

ii. From the optimal tableau above, determine the number of acres that should be devoted to growing each type of crop, and the corresponding profit. 3

iii. There is a glut of barley on the market, so its price falls substantially. By how much can the profit on barley fall before the optimal solution moves to a different corner point? 11

4. Consider the problem in section 3.4.1 of the lecture notes dealing with an equality constraint. We have $x_1 + x_2 + x_3 = 1$ and so we substitute $x_3 = 1 - x_1 - x_2$ in the objective function and the constraints to give us the problem:

$$\textbf{Maximise} \quad z = -4 + x_1 + 2x_2$$

$$\textbf{subject to} \quad \begin{cases} 4x_1 + 3x_2 + x_4 = 3.6 \\ 2x_1 + 5x_2 + x_5 = 4 \end{cases}$$

$$\textbf{and} \quad x_j \geq 0, j = 1, 2, 4, 5.$$

An initial basic feasible solution is $x_1 = x_2 = 0$, $x_4 = 3.6$ and $x_5 = 4.0$. Thus the initial simplex tableau is

Basic	z	x_1	x_2	x_4	x_5	Solution
x_4	0	4	3	1	0	$\frac{18}{5}$
x_5	0	2	5	0	1	4
z	1	-1	-2	0	0	-4

By using the Simplex tableau calculations demonstrate that there is an error in the reasoning, find and correct the error and then use the Simplex method to find the correct optimum solution. 20