

ASSIGNMENT 1, MAT1030: CALCULUS

We advise you to solve all the questions given below. However please hand in the solutions of the **EVEN** questions only at the end of our lecture on Thursday the 24/10/2013. The even questions will be marked and returned to you with appropriate feedback.

PLEASE do not forget to write your **Name** and **Surname** and the **Surname** of your Personal Tutor on your solutions. Please staple together all your working pages, otherwise some page/s may get lost or mixed up with other pages of other people, simply generating a complete mess. Thank you.

1. State the maximal domains of the functions

$$(a) f : x \mapsto \sqrt{x^2 - 9}, \quad (b) g : x \mapsto \frac{3x + 1}{x^2 - 2x - 3}.$$

2. Find the range of the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2 - 4x - 3$. Why does f not have an inverse? How can the domain and codomain of f be restricted so that the resulting function *does* have an inverse?

3. State the values of $\operatorname{sgn}(-3)$, $|- \pi|$, $\lfloor \sqrt{2} \rfloor$, $\sqrt{(-1)^2}$.

4. Sketch graphs of (i) $\operatorname{sgn}(1-x)$, (ii) $x - \lfloor x \rfloor$, (iii) $\operatorname{sgn}(\sin x)$ (iv) $H(x-2)x^2$

5. If $f : (-\infty, 0) \rightarrow (1, \infty)$ by $x \mapsto 1 - 5x$ and $g : [1, \infty) \rightarrow [0, \infty)$ by $x \mapsto \sqrt{x-1}$, define (if they exist) the functions f^{-1} , g^{-1} , $f \circ g$ and $g \circ f$.

6. For each of the following functions with maximal domain, and codomain \mathbb{R} , state whether it is even, odd, periodic, one-to-one, onto. Find and simplify the composite functions $f \circ f$, $f \circ g$, $g \circ f$, $g \circ h$, and state their domains.

$$(a) f : x \mapsto x^2 + 1, \quad (b) g : x \mapsto \frac{2x}{x-2}, \quad (c) h : x \mapsto \tan 2x,$$

7. If $f(x) = \frac{2x+1}{x^2+2}$ for $x \in \mathbb{R}$, find the set of values of y for which $f(x) = y$ has real roots for x . Hence state the range of f .

8. Sketch a graph of

$$f(x) = \begin{cases} 2x, & -1 \leq x < 0 \\ x^2, & 0 \leq x \leq 1. \end{cases}$$

Define the inverse function.

9. Show that if $f \circ g$ is invertible, then $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

10. Express each of the following in partial fractions:

$$(a) \frac{4x-3}{(x+1)(x^2+x+1)}$$

$$(b) \frac{2x}{(x-5)^2(x+1)}$$

$$(c) \frac{x^3+1}{x^2+7x+12}$$

$$(d) \frac{3x+7}{(x+1)^2(x+3)}$$

11. Express each of the following in partial fractions:

$$(a) \frac{x^2 + 2}{4x^5 + 4x^3 + x} \quad (b) \frac{3x^4 + 6x^3 - 2x^2 + 4}{x^3 + 2x^2}$$

12. The functions f and g are defined as follows

$$f(x) = \begin{cases} x^2 + 4 & x \geq 1 \\ x & x < 1 \end{cases} \quad g(x) = \begin{cases} 3x & x \geq 0 \\ -2x & x < 0. \end{cases}$$

Find $f \circ g$.

13. Assume that f is an odd function and g is an even function, both defined on the real line. Is each of the following functions odd, even or neither?

$$f \circ g, \quad g \circ f, \quad f \circ f, \quad g \circ g.$$

14. Find all the real values of the constants A and B such that $f(x) = Ax + B$ satisfies (i) $f \circ f(x) = f(x)$ for all x , (ii) $f \circ f(x) = x$ for all x .

15. For what values of a, b and c is the function $f(x) = \frac{x - a}{bx - c}$ self inverse?

16. Show that if $f(x)$ is an odd function, its derivative is an even function. What can you say about $f^{(n)}(x)$?