

## PDEs: UN-ASSESSED HOMEWORK II

We advise you to solve ALL the questions below.

**However, please hand in only the solutions of the starred questions at the end of our lectures on Thursday 20/3/2014.**

These questions will be marked and returned to you with appropriate feedback.

Thought of the homework:

To some-One who could view the universe from a unified standpoint, the entire creation would appear as a unique truth and necessity.

D'Alembert, L'Encyclopédie (1751)

For each of the following second order linear partial differential equations:

- Classify the equation as hyperbolic, parabolic or elliptic.
- Reduce it to canonical form.
- Solve (when possible) the canonical equation.
- Find (when possible) the general solution.

a\*)  $u_{xx} - 4u_{xy} + 4u_{yy} + u_x + u_y = 6x - y.$

b)  $u_{xx} - 8u_{xy} + 12u_{yy} + u_x + u_y = 6x - y.$

c\*)  $u_{xx} + 8u_{xy} + 4u_{yy} = 0.$

d\*)  $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0.$

e\*)  $x^2u_{xx} - y^2u_{yy} = xy^3, \quad x > 0, \quad y > 0.$

f)  $u_{xx} - u_{yy} = (x + y)^2.$

g) Transform Tricomi's equation

$$yu_{xx} + u_{yy} = 0$$

into canonical form for  $y < 0.$

h)  $u_{xx} - 4u_{xy} + 4u_{yy} + u_x + u_y = 0.$