PDEs: UN-ASSESSED HOMEWORK III
Please solve all the questions below
Please hand (the solutions of) it in at the end of our second lecture on Thursday 3/4/2014

1) Solve the heat equation on bounded intervals in each of the problems below, with the given initial and boundary conditions:

(a) $u_{xx} = u_t, \quad u(0,t) = u(1,t) = 0, \ \ t \geq 0; \quad u(x,0) = \sin(\pi x), \ \ 0 \leq x \leq 1.$

(b) $ku_{xx} = u_t, \quad u_x(0,t) = u_x(4,t) = 0, \ \ t \geq 0; \quad u(x,0) = x^2, \ \ 0 \leq x \leq 4;$

2) Solve the heat equation on unbounded intervals with the given initial and boundary conditions:

(a) $-ku_{xx} + u_t = 0, \ \ x \geq 0, \quad u(0,t) = 0, \ \ t \geq 0,$

$$u(x,0) = \begin{cases} 1, & 0 < x \leq 1, \\ 0, & x > 1. \end{cases}$$

(b) $-ku_{xx} + u_t = 0, \ \ x \geq 0, \quad u(0,t) = 0, \ \ t \geq 0, \ \ u(x,0) = e^{-\alpha x}, \ \ \alpha > 0.$

3) Suppose u is a solution of the heat equation

$$-ku_{xx} + u_t = 0$$

for $0 < x < L$, $t > 0$. Let $0 < a < b < L$. Prove that, for any positive t,

$$\frac{d}{dt}\int_a^b \frac{1}{2}u(x,t)^2 dx = k[uu_x]_a^b - \int_a^b ku_x(x,t)^2 dx.$$