MAT2011: LINEAR PDEs. 2014 - MOCK TEST

You have 60 minutes.

Question 1. [25]

Consider the first order linear homogeneous partial differential equation

$$2u_x - 3u_y = 7u.$$

Denote its characteristic curves by $\xi = \xi(x, y),\ \eta = \eta(x, y)$.

- By choosing $\xi = x$, find the characteristic curve for $\eta = \eta(x, y)$.
- Find its general solution.
- Hence find the particular solution satisfying $u(x, y = \frac{2x}{3}) = e^{3x}$.

Question 2. [35]

Consider the following second order linear non-homogeneous partial differential equations

$$u_{xx} - u_{yy} = 4(x + y)^3.$$

- Classify the equation as hyperbolic, parabolic or elliptic.
- Reduce it to canonical form.
- By solving the canonical equation find its general solution.

Question 3. [40]

By using the separation of variables method, solve the heat equation with the given boundary and initial condition:

$$u_t = u_{xx}, \quad 0 < x < 1, \quad t \geq 0,$$
$$u(0, t) = u(1, t) = 0, \quad t \geq 0,$$
$$u(x, 0) = \sin(7\pi x).$$