

MAT2011: LINEAR PDEs. ASSESSED TEST. MAY 2014.

Write your answers in the answer book provided. **Write (in capital letters and clearly please) your name on the outside cover.** Please do not seal the answer book flap as it is not necessary. Thanks. Do any rough working you need in the answer book, but indicate clearly that it is rough working. You should show sufficient working to demonstrate the method you have used in arriving at your solution. This is a *closed book* test.

You have 60 minutes.

Question 1.

[25]

Consider the first order linear homogeneous partial differential equation

$$u_x - 2u_y = u.$$

Denote its characteristic curves by $\xi = \xi(x, y)$, $\eta = \eta(x, y)$.

- By choosing $\xi = x$, find the characteristic curve for $\eta = \eta(x, y)$.
- Find its general solution.
- Hence find the particular solution satisfying $u(0, y) = y$.

Question 2.

[35]

Consider the following second order linear non-homogeneous partial differential equations

$$u_{xx} - u_{yy} = (x + y)^2.$$

- Classify the equation as hyperbolic, parabolic or elliptic.
- Reduce it to canonical form.
- By solving the the canonical equation find its general solution.

Question 3.

[40]

By using the separation of variables method, solve the heat equation with the given boundary and initial condition:

$$\begin{aligned}u_t &= u_{xx}, \quad 0 < x < 1, \quad t \geq 0, \\u(0, t) &= u(1, t) = 0, \quad t \geq 0, \\u(x, 0) &= \sin(2\pi x).\end{aligned}$$