MAT3008 and MMath3031: LAGRANGIAN AND HAMILTONIAN DYNAMICS

ASSESSED COURSEWORK WORTH 10 POINTS OF THE TOTAL EXAM MARK

Please hand in the solutions of the all the questions by Tuesday 30/11/2010

Thought of the coursework:
Who by a vigour of mind almost divine, the motions and figures of planets, the paths of comets, and the tides of the sea, first demonstrated. Newton's Epitaph.

1) A simple pendulum comprises of a particle of mass m attached to the end of a light stiff rod of length l. The pendulum is in a constant gravitational field, with gravity acting downwards. The pivot P executes a uniform circular motion witn frequency ω along a horizontal cirlce of constant radius a.

a) Find the Lagrangian for the above system.

b) Given that two Lagrangians L and $\overline{L}$ are said to be equivalent if they are related by

$$\overline{L}(\dot{q}_i(t), q_i(t), t) = L(\dot{q}_i(t), q_i(t), t) + \frac{dF}{dt}(q_i(t), t),$$

where $F(q_i, t)$ is an arbitrary differentiable function of the generalised coordinates q_i and time t, find an equivalent Lagrangian for the above system.

c) Hence find the Lagrangian equations of motion from the equivalent Lagrangian $\overline{L}$.

2) Show that the transformation
$Q = \ln\left(\frac{\sin p}{q}\right)$
$P = q \cot p$
is canonical/symplectic. Hence find the generating function $F_1(q, Q)$.

3) The coordinates of a canonical/symplectic transformation $(q, p) \to (Q, P)$ are given by

$Q = \tan^{-1}(\frac{q}{p})$

$P = \frac{q^2+p^2}{2} + \frac{q}{p}$.

Apply this transformation to transform the Hamiltonian

$$H(q, p) = \frac{q^2 + p^2}{2}$$

into the new Hamiltonian

$$K(Q, P) = H\left(q(Q, P), p(Q, P)\right).$$

Hence solve the problem in the new coordinates (Q, P) and then solve for (q, p) in terms of the time. Assume that the initial conditions are
$q(t = 0) = q_0 > 0, \quad p(t = 0) = 0.$