**MAT3008: LAGRANGIAN AND HAMILTONIAN DYNAMICS**
UN-ASSESSED COURSEWORK III
We politely but strongly advise you to solve ALL the questions below.
**However, please hand in only the solutions of the starred questions at the end of our second lecture on Tuesday 30/04/2013.**
These questions will be marked and returned to you with appropriate feedback.

Thought of the coursework:
The laws of Nature are written in the language of mathematics ... the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word.
Galileo Galilei. "Dialogue Concerning the Two Chief Systems of the World" (1632).

1) A pendulum comprises of a particle of mass $m$ attached to the end of a light (massless) stiff rod of length $l$. The pendulum is in a constant gravitational field, with gravity acting downwards. The massless pivot $P$ executes a uniform circular motion witn frequency $\omega$ along a horizontal cirlce of constant radius $a$.

a) Find the Lagrangian for the above system.

b) Given that two Lagrangians $L$ and $\overline{L}$ are said to be equivalent if they are related by

$$\overline{L}(\dot{q}_i(t), q_i(t), t) = L(\dot{q}_i(t), q_i(t), t) + \frac{dF}{dt}(q_i(t), t),$$

where $F(q_i, t)$ is an arbitrary differentiable function of the generalised coordinates $q_i$ and time $t$, find an equivalent Lagrangian for the above system.

c) Hence find the Lagrangian equations of motion from the equivalent Lagrangian $L$.

2*) Find (if any) all the values of $\alpha$ and $\beta$ such that the transformation

$$Q = \beta\sqrt{p}\cos q, \quad P = -\alpha\sqrt{p}\sin q$$

is symplectic.

3*) Show that generally the Poisson bracket $[\phi, M_z] \neq 0$ for differentiable scalar functions of the form $\phi = \phi(x, y, z)$.

4*) Show that the transformation $F(p, Q) = -p(e^Q - 1)$ is canonical and transform the Hamiltonian

$$H = (1+q)^2\frac{p^2}{2} + \log(1+q)$$

into the new Hamiltonian $K(P, Q)$.

5*) Consider the transformation $Q = \dfrac{p^2}{4q}$, $P = -\dfrac{4q^2}{3p}$. Show that the transformation is canonical. Further consider the Lagrangian $L = q\dot{q}^2$; find the corresponding Hamiltonian. Hence transform it into the new Hamiltonian $K(P, Q)$.