MAT3008: LAGRANGIAN AND HAMILTONIAN DYNAMICS. ASSESSED TEST, May 2013

Write your answers in the answer book provided. **Write (in capital letters and clearly please) your name on the outside cover.** Do any rough working you need in the answer book, but indicate clearly that it is rough working. You should show sufficient working to demonstrate the method you have used in arriving at your solution. This is a *closed book* test.

You have 60 minutes.

1) A particle of mass $m > 0$ moves on a vertical plane under the action of gravity of intensity g, along a smooth curve of the form $f(x) = e^{x^2}$. Find its Lagrangian.

2) Two Lagrangians L and $\overline{L}$ are said to be equivalent if they are related by

$$\overline{L}(\dot{q}_i, q_i, t) = L(\dot{q}_i, q_i, t) + \frac{dF}{dt}(q_i, t), \quad i = 1, 2, ...n;$$

$F(q_i, t)$ is an arbitrary differentiable function of the generalised coordinates q_i and time t. Prove that L and $\overline{L}$ have the same Lagrangian equations of motion and hence they describe the same motion.

3) A particle of unit mass moves under the action of the force generated by a central field having potential energy $V = V(r(t))$, where $r(t) \geq 0$ is the distance of the particle from the fixed centre of force which we take as the origin O of our system of coordinates.

a) Show that the motion is planar.

b) By using polar coordinates (r, θ) on the plane of motion, find the total energy E of the particle.

c) Take the particular case in which the above particle moves in a potential having the form $V(r) = \alpha r^4$ with $\alpha > 0$ and constant.

i) By using the effective potential energy theory, discuss the possible type of motion for the particle in the above potential.

ii) Find the minimum value $U(r_0)$ of the effective potential, where r_0 is such that $E_0 = U(r_0)$ and therefore $\dot{r} = 0$ for $E = E_0$.

iii) Thus show that for $E = E_0 = U(r_0)$ the particle moves of uniform motion on a circle about the origin and find the period of its motion.