

**Name:**

**Tutor:**

**Please hand (the solutions of) this in at the end of our second lecture on Thursday the 13/11/2008**

**Question 1:** By using the Fundamental Theorem of Arithmetic, show that if a positive integer  $m$  is not a perfect square, (namely if it is not of the form  $m = n^2$ , where  $n$  is an integer), then  $\sqrt{m}$  is irrational.

**Question 2:** Given positive integers  $a, b$  their product is a multiple of both and therefore they have a *least common multiple* usually denoted by  $l(a, b)$ . By denoting by  $g(a, b)$  their *highest common factor*, prove that  $l(a, b) \cdot g(a, b) = ab$ . *Hint: Use the Fundamental Theorem of Arithmetic.*

**Question 3:** Show by induction that for every natural number  $n$  the sum of the first  $n$  odd numbers is equal to  $n^2$ .

**Question 4:** Find the solutions within the set of natural numbers of the Diophantine equation  $11x - 7y = 3$ . Does the Diophantine equation  $15x - 5y = 2$  possess solutions within the set of natural numbers? Give reasons for your answer.