ASSIGNMENT 3, MAT1017: PROOF, PROBABILITY and EXPERIMENT, 2008

Name: **Tutor:**

Please hand (the solutions of) this in at the end of our second lecture on Thursday 27/11/2008

Question 1: Show by contraposition that if $a \geq 2$ and $a^m + 1$ is a prime number, with m any natural number, then a must be even.

Question 2: If $m > 1$ and $a^m - 1$ is prime then $a = 2$ and m is prime, that is $2^p - 1$ with p a prime is a Mersenne prime.

Question 3: Use the theory of congruences to show that the polynomials given below have no integer roots:
(a) $x^3 + x^2 - x + 3$;
(b) $x^3 - x^2 - x + 11$.

Question 4: Where it exists, find the general solution of the linear congruences (give detailed reasons for your answers):

- $10x \equiv 6 \pmod{14}$;
- $7x \equiv 2 \pmod{9}$;
- $9x \equiv 7 \pmod{6}$.

Question 5: By using Fermat's Little Theorem:
(a) Show that 6 is the least non-negative residue of $2^{68} \pmod{19}$, that is, show that $2^{68} \equiv 6 \pmod{19}$.
(b) Find the least non-negative residue of $3^{91} \pmod{23}$.