MAT1017: ASSESSED TEST - 31/10/2008

Question 1: Show by induction that for every natural number n, the sum

$$\sum_{k=1}^{k=n} k^3 = \frac{n^2(n+1)^2}{4}.$$

Question 2: Given a and b integers with $b > 0$. Then there is a pair of integers q and r such that $a = qb + r$, with $0 \leq r < b$. Prove that the pair (q, r) is unique.

Question 3: Assume that the greatest common divisor of the two integers a and b is $g(a, b) = 1$.

(i) Show that if $a|c$ and $b|c$ than $ab|c$;

(ii) If $a|bc$ then $a|c$.

Question 4: Show that for each prime $p \geq 5$ the natural number $p^2 + 2$ is divisible by 3 and therefore it is a composite number. *Hint: Let* $p = 3q + r, \quad r = 0, 1, 2;$ however the case $r = 0$ must be excluded (why?).

Question 5: Show by induction that

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} \geq \sqrt{n}.$$