

ASSIGNMENT 1, MAT1026: PROOF, 2009

Name:

Tutor:

Please hand (the solutions of) this in at the end of our second lecture on Thursday the 29/10/2009

Question 1: Prove (by contradiction) that the polynomial $f(x) = x^4 + 2x^2 + 2x + 1$ cannot be written as the product of two quadratic polynomials with integer coefficients.

Question 2: Prove (by contradiction) that at a party of at least two people, there are at least two who have the same number of friends at the party.

Question 3: If a and b are positive integers, prove by contraposition that $ax^2 + bx + (b - a) = 0$ has no positive integer root.

Question 4: Prove by contradiction that the sum of the squares of three consecutive integers cannot leave remainder -1 on division by 12.

Question 5: Show that the statements P and $\sim\sim P$ (read not not P) are the same. Further suppose that P is a statement from which you can deduce $\sim P$ (read not P). Which (if any) of the following conclusions can you draw:

- (i) P is true;
- (ii) P is false;
- (iii) $\sim P$ is true;
- (iv) $\sim P$ is false.