

Please hand (the solutions of) this in at the end of our second lecture on Tuesday the 2/11/2010

Please do not forget to write your **Name** and **Surname** and the **Surname** of your Personal Tutor on your solutions. Please staple together all your working pages, otherwise some page/s may get lost or mixed up with other pages of other people, simply generating a fine mess. Thank you!

Question 1: Prove (by contradiction) that the polynomial $f(x) = x^4 + 2x^2 + 2x + 1$ cannot be written as the product of two quadratic polynomials with integer coefficients.

Question 2: If a and b are positive integers, prove that $ax^2 + bx + (b - a) = 0$ has no positive integer root.

Question 3: Prove by contradiction that the sum of the squares of three consecutive integers cannot leave remainder -1 on division by 12.

Question 4: Prove by contradiction that if p is a prime dividing a product ab of integers, then either $p|a$ or $p|b$ or both.

Question 5: Show by induction that for every natural number n the sum of the first n odd numbers is equal to n^2 .

Question 6: Show by contraposition that if $a \geq 2$ and $a^m + 1$ is a prime number, with m any natural number, then a must be even.