UN-ASSESSED ASSIGNMENT 2, MAT1026: PROOF, 2010

Please hand (the solutions of) this in at the end of our second lecture on Tuesday the 16/11/2010
Please do not forget to write your **Name** and **Surname** and the **Surname** of your Personal Tutor on your solutions. Please staple together all your working pages, otherwise some page/s may get lost or mixed up with other pages of other people, simply generating a fine mess. Thank you!

Question 1: Use Euclid's algorithm to calculate the highest common factor g of the numbers (89,55), and of the numbers (3132,7200). For the numbers (89,55) find integers x and y such that $g = 55x + 89y$.

Question 2: Given positive integers a, b their product is a multiple of both and therefore they have a *least common multiple* usually denoted by $l(a, b)$. Assuming $l(a, b)$ known, and by denoting by $g(a, b)$ their *highest common factor*, prove that $l(a, b) \cdot g(a, b) = ab$.

Question 3: Find the solutions within the set of natural numbers of the Diophantine equation $11x - 7y = 3$. Does the Diophantine equation $15x - 5y = 2$ possess solutions within the set of natural numbers? Give reasons for your answer.

Question 4: Show by contraposition that if $a \geq 2$ and $a^m + 1$ is a prime number, with m any natural number, then a must be even.

Question 5: If $m > 1$ and $a^m - 1$ is prime then show that $a = 2$ and m is prime. [That is $a^m - 1$ has the form $2^p - 1$ with p a prime which is a Mersenne prime].