UN-ASSESSED ASSIGNMENT 3, MAT1026: PROOF, 2010

Please hand (the solutions of) this in at the end of our second lecture on Tuesday the 30/11/2010
Please do not forget to write your **Name** and **Surname** and the **Surname** of your Personal Tutor on your solutions. Please staple together all your working pages, otherwise some page/s may get lost or mixed up with other pages of other people, simply generating a fine mess. Thank you!

Question 1: Show by contraposition that if $a \geq 2$ and $a^m + 1$ is a prime number, with m any natural number, then a must be even.

Question 2: Use the theory of congruences to show that the polynomials given below have no integer roots:
(a) $x^3 + x^2 - x + 3$;
(b) $x^3 - x^2 - x + 11$.

Question 3: Where it exists, find the general solution of the linear congruences (give detailed reasons for your answers):

a) $10x \equiv 6 \pmod{14}$;

b) $7x \equiv 2 \pmod{9}$;

c) $9x \equiv 7 \pmod{6}$.

Question 4: By using Fermat's Little Theorem:
(a) Show that 6 is the least non-negative residue of $2^{68} \pmod{19}$, that is, show that $2^{68} \equiv 6 \pmod{19}$.
(b) Find the least non-negative residue of $3^{91} \pmod{23}$.