MAT3008 and MMath3031: LAGRANGIAN AND HAMILTONIAN DYNAMICS

UN-ASSESSED COURSEWORK I

We politely but strongly advise you to solve ALL the questions below

However, please hand in only the solutions of the starred questions at the end of our lecture on Tuesday 8/11/2011

These questions will be marked and returned to you with appropriate feedback

Thought of the coursework:

The full significance of Lagrange's ideas has yet to be revealed. We shall see that such formulas have proved to be the most valuable knowledge we have ever acquired about nature. For example, the remarkable invariance of the Lagrangian equation of motion with respect to arbitrary point transformations, gives these equations a unique position in the development of mathematical thought.

Cornelius Lanczos, The Variational Principles of Mechanics

1*) A particle of mass m is constrained to move in a vertical plane under the influence of gravity along a smooth given curve with parametric equations $z = h(\theta)$, $x = f(\theta)$, where the $z-$axis is vertically upward. Taking θ as the generalised coordinate show that the Lagrangian is

$$L = \frac{m}{2}\dot{\theta}^2 \left[\left(\frac{dh}{d\theta}\right)^2 + \left(\frac{df}{d\theta}\right)^2 \right] - mgh(\theta).$$

Hence find the Lagrangian L for the particular case $z = l\theta, \quad x = l\cos\theta$, where l is constant.

2*) A simple pendulum comprises of a particle of mass m attached to the end of a light stiff rod of length l. The simple pendulum is in a constant gravitational field, with gravity acting downwards. The massless pivot P executes a clockwise circular motion on a circumference with centre O and radius a with constant angular velocity ω in a horizontal plane, so that at time t, OP makes an angle ωt with the horizontal positive x-axis (say).

1. Find the Lagrangian for the motion of the pendulum.
2. Find an equivalent Lagrangian $\overline{L}$ for the system.
3. From $\overline{L}$, find the Lagrangian equation of motion.

3) Find the Lagrangian for a pendulum with variable length. Consider the case when $r(t) = a + b\cos(\omega t)$, with $a >> 2b > 0$; substitute in the Lagrangian and

then find the equation of motion for the angle variable. Hence obtain a differential equation of Hill-Mathieu type by linearizing the system under the assumption that the angle with the downward drawn vertical remains small throughout the motion (neglect all the terms of higher order including $\dot{\theta}^2$ and terms containing products of the form ab).

4) Find the Lagrangian for a system of two particles of mass m_1 and m_2, attached to a string of constant length l. The two particles move on the opposite sides of a smooth vertical plane in the gravity field.

5*) Find the Lagrangian and then the equations of motion for a particle of unit mass moving in the potential $V(r) = \alpha r^4, \quad \alpha > 0$. By using the effective potential energy, discuss the possible type of motion for the particle in the above potential. Find also the angular velocity $\dot{\theta} = \omega$ in a circular orbit of radius $r = r_0 = constant$. Then find the period of the motion of the particle in this circular orbit.