LAGRANGIAN AND HAMILTONIAN DYNAMICS: ASSESSED COURSE-WORK II

Please hand (the solutions of) this in at the end of our lecture on Friday 7/11/2008

Thoughts of the coursework:

1) Who by a vigour of mind almost divine, the motions and figures of planets, the paths of comets, and the tides of the sea, first demonstrated. Newton's Epitaph.

2) The full significance of Lagrange's ideas has yet to be revealed. We shall see that such formulas have proved to be the most valuable knowledge we have ever acquired about nature. For example, the remarkable invariance of the Lagrangian equation of motion with respect to arbitrary point transformations, gives these equations a unique position in the development of mathematical thought.

Cornelius Lanczos: The Variational Principles of Mechanics

1) Show that the transformation

$$Q = q\cos\alpha - p\sin\alpha$$

$$P = q\sin\alpha + p\cos\alpha,$$

α being a constant, is symplectic.

2) Find (if any) all the values of α and β such that the transformation

$$Q = \beta\sqrt{p}\cos q, \quad P = -\alpha\sqrt{p}\sin q$$

is symplectic.

3) Show that generally the Poisson bracket $[\phi, M_z] \neq 0$ for differentiable scalar functions of the form $\phi = \phi(x, y, z)$.

4) Show that the transformation $F(p, Q) = -p(e^Q - 1)$ is canonical and transform the Hamiltonian

$$H = (1+q)^2\frac{p^2}{2} + \log(1+q)$$

into the new Hamiltonian $K(P, Q)$.

5) Consider the transformation $Q = \dfrac{p^2}{4q}$, $P = -\dfrac{4q^2}{3p}$. Show that the transformation is canonical. Further consider the Lagrangian $L = q\dot{q}^2$; find the corresponding Hamiltonian. Hence transform it into the new Hamiltonian $K(P, Q)$.