**MAT3008 and MMath3031: LAGRANGIAN AND HAMILTONIAN DYNAMICS**

UN-ASSESSED COURSEWORK I

We politely but strongly advise you to solve ALL the questions below

**However, please hand in only the solutions of the starred questions by Tuesday 26/10/2010**

These questions will be marked and returned to you with appropriate feedback

The solutions to ALL the questions will be up-loaded on the module web-page in due course

Thought of the coursework. To some-One who could view the universe from a unified standpoint, the entire creation would appear as a unique truth and necessity.

D'Alembert, L'Encyclopédie (1751)

* 1) A particle of mass $m$ is constrained to move in a vertical plane under the influence of gravity along a smooth given curve with parametric equations $z = h(\theta)$, $x = f(\theta)$, where the $z-$axis is vertically upward. Taking $\theta$ as the generalised coordinate show that the Lagrangian is

$$L = \frac{m}{2}\dot{\theta}^2 \left[ \left( \frac{dh}{d\theta} \right)^2 + \left( \frac{df}{d\theta} \right)^2 \right] - mgh(\theta).$$

Hence find the Lagrangian $L$ for the particular case $z = l\theta$, $\quad x = l\cos\theta$, where $l$ is constant.

* 2) A simple pendulum comprises of a particle of mass $m$ attached to the end of a light stiff rod of length $l$. The simple pendulum is in a constant gravitational field, with gravity acting downwards. The massless pivot $P$ executes an anticlockwise circular motion on a circumference with centre $O$ and radius $a$ with constant angular velocity $\omega$ in a vertical plane, so that at time $t$, $OP$ makes an angle $\omega t$ with the downward drawn vertical. Hence the displacement of the pivot $P$ is

$$x_p = a\sin\omega t, \quad y_p = a\cos\omega t,$$

where $x_p$ and $y_p$ are its cartesian coordinates, relative to axes $(Ox, Oy)$ with the $Oy$ direction vertically downwards.

i) Show that the Lagrangian for the motion of the simple pendulum is

$$L = \frac{ma^2}{2}\omega^2 + \frac{ml^2}{2}\dot{\theta}^2 + mga\cos(\omega t) + mgl\cos\theta + mla\omega\dot{\theta}\cos(\theta - \omega t),$$

where $\theta$ is the angle made by the stiff rod with the downward vertical, $g$ is the constant acceleration due to gravity, and the dots denote differentiation with respect to time.

ii) Show that an equivalent Lagrangian for the system is given by

$$\overline{L} = \frac{ml^2}{2}\dot{\theta}^2 + mla\omega^2\cos(\theta - \omega t) + mgl\cos\theta.$$

iii) From $\overline{L}$, find the Lagrangian equation of motion.

iv) Assume now that $\omega$ and $\theta$ remain small throughout the motion of the pendulum, so that, in the equation of motion, we can neglect all the terms of order $\omega^2\theta$ or smaller. Linearize the system under this assumption and find its general solution under the hypothesis $\frac{g}{l} \neq \omega^2$.

3) A massless non-extensible string passes over a small pulley and carries at one end a mass $2m$ and beneath it, supported by a spring with spring constant $k > 0$, a second mass $m$. On the other end there is a mass $m$, and beneath it, supported by a spring with spring constant $k > 0$, a second mass $m$.

i) Find the Lagrangian of this system.

ii) Find the corresponding Lagrangian equations of motion.

4) Find the Lagrangian for a system of two particles of mass $m_1$ and $m_2$, attached to a string of constant length $l$. The two particles move on the opposite sides of a smooth vertical plane in the gravity field.

* 5) Find the Lagrangian $L$ for a pendulum which is constrained to move on a vertical plane under the influence of gravity, and such that its length $r(t)$ varies in time as $r(t) = a + b\sin(\omega t)$ where $a > b > 0$. Find Lagrange's equation of motion for the Lagrangian coordinate $\theta$.