MAT3008 and MMath3031: LAGRANGIAN AND HAMILTONIAN DYNAMICS

UN-ASSESSED COURSEWORK II

We politely but strongly advise you to solve ALL the questions below

However, please hand in only the solutions of the starred questions by Tuesday 16/11/2010

These questions will be marked and returned to you with appropriate feedback

The solutions to ALL the questions will be up-loaded on the module web-page in due course

Thought of the coursework:

The full significance of Lagrange's ideas has yet to be revealed. We shall see that such formulas have proved to be the most valuable knowledge we have ever acquired about nature. For example, the remarkable invariance of the Lagrangian equation of motion with respect to arbitrary point transformations, gives these equations a unique position in the development of mathematical thought.

Cornelius Lanczos: The Variational Principles of Mechanics

1*) The potential energy of a particle of unit mass is $V(r)$. Show that the motion is planar. By using polar coordinates (r, θ) on the plane of motion, find the total energy of the particle. Show the magnitude of its angular momentum is given by

$$J = r^2\dot{\theta}.$$

By using $J = r^2\dot{\theta}$ show that the energy takes the form

$$E = \frac{1}{2}\dot{r}^2 + \frac{J^2}{2r^2} + V(r) = \frac{1}{2}\dot{r}^2 + U(r),$$

where $U(r)$ is the effective potential. Take the particular case in which the above particle moves in a potential having the form

a) $V(r) = \alpha r^4, \qquad \alpha > 0;$

b) $V(r) = -\dfrac{\gamma}{r^n}, \qquad \gamma > 0, \qquad n \geq 1.$

By using the effective potential energy theory, discuss the possible type of motion for the particle in the above potentials.

2*) Find (if any) all the values of α and β such that the transformation

$$Q = \beta\sqrt{p}\cos q, \quad P = -\alpha\sqrt{p}\sin q$$

is symplectic.

3*) Show that generally the Poisson bracket $[\phi, M_z] \neq 0$ for differentiable scalar functions of the form $\phi = \phi(x, y, z)$.

4) Show that the transformation $F(p, Q) = -p(e^Q - 1)$ is canonical and transform the Hamiltonian

$$H = (1+q)^2 \frac{p^2}{2} + \log(1+q)$$

into the new Hamiltonian $K(P, Q)$.

5*) Consider the transformation $Q = \dfrac{p^2}{4q}$, $P = -\dfrac{4q^2}{3p}$. Show that the transformation is canonical. Further consider the Lagrangian $L = q\dot{q}^2$; find the corresponding Hamiltonian. Hence transform it into the new Hamiltonian $K(P, Q)$.