MAT3008 and MMath3031: LAGRANGIAN AND HAMILTONIAN DYNAMICS

UN-ASSESSED COURSEWORK II

We politely but strongly advise you to solve ALL the questions below.

However, please hand in only the solutions of the starred questions by Tuesday 22/11/2011.

These questions will be marked and returned to you with appropriate feedback.

Thoughts of the coursework:

To some-One who could view the universe from a unified standpoint, the entire creation would appear as a unique truth and necessity.

D'Alembert, L'Encyclopédie (1751)

Who by a vigour of mind almost divine, the motions and figures of planets, the paths of comets, and the tides of the sea, first demonstrated.

Newton's Epitaph.

1) The potential energy of a particle of unit mass is $V(r)$. Show that the motion is planar. By using polar coordinates (r, θ) on the plane of motion, find the total energy of the particle. Take the particular case in which the above particle moves in a potential having the form $V(r) = -\frac{\gamma}{r^n}, \quad \gamma > 0, \quad n \geq 1$. By using the effective potential energy theory, discuss the possible type of motion for the particle in the above potential.

2*) Find (if any) all the values of α and β such that the transformation

$$Q = \beta\sqrt{p}\cos q, \quad P = -\alpha\sqrt{p}\sin q$$

is symplectic.

3*) Show that generally the Poisson bracket $[\phi, M_z] \neq 0$ for differentiable scalar functions of the form $\phi = \phi(x, y, z)$.

4) Show that the transformation $F(p, Q) = -p(e^Q - 1)$ is canonical and transform the Hamiltonian

$$H = (1+q)^2\frac{p^2}{2} + \log(1+q)$$

into the new Hamiltonian $K(P, Q)$.

5*) Consider the transformation $Q = \frac{p^2}{4q}, \ P = -\frac{4q^2}{3p}$. Show that the transformation is canonical. Further consider the Lagrangian $L = q\dot{q}^2$; find the corresponding Hamiltonian. Hence transform it into the new Hamiltonian $K(P, Q)$.