Numerical Solutions for PDEs
CLASS TEST
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Exercise 1:
We consider the following scheme

$$U_{j,n+1} = U_{j,n} + r\delta_x^2 U_{j,n} + 2r\delta_x^2 U_{j,n+1},$$

where $r = \frac{\Delta t}{\Delta x^2}$.

1. Show that this scheme is consistent with the PDE $u_t = 3u_{xx}$. [7pt]

2. Carry out a Von Neumann stability analysis and find under what restriction on r the scheme is stable. [8pt]

3. Describe an algorithm to find an approximate solution to the problem

$$\begin{cases} u_t = 3u_{xx}, \quad x \in [0,1], t > 0 \\ u(0,t) = 0, \\ u_x(1,t) = g(t), \\ u(x,0) = f(x), \end{cases}$$

using the scheme given above. [5pt]

Exercise 2:
We consider the first order wave equation

$$u_t + au_x = 0,$$

together with the FTFS scheme defined by

$$\frac{F_t U_{j,n}}{\Delta t} + a\frac{F_x U_{j,n}}{\Delta x} = 0.$$

1. Work out the local truncation error for this scheme and show that the scheme is consistent with the PDE. What happens when $p = a\Delta t/\Delta x = -1$? [7pt]

2. Under what restriction on p is the scheme stable. [8pt]