Numerical Solutions for PDEs: Coursework to be handed in on Tuesday the 6th of March

Exercise 1:
The central time central space (CTCS) operator for the diffusion equation is defined by

$$\mathcal{L}_\Delta = \frac{\delta_t}{\Delta t} - \frac{\delta_x^2}{\Delta x^2}.$$

Work out the local trunctation error and show that the CTCS scheme is consistant with the diffusion equation.

Exercise 2:
Under what restriction the following scheme is Von Neumann stable:

$$U_{j,n+1} - U_{j,n-1} = 2r\delta_x^2 U_{j,n},$$

where U is an approximation for $u(x,t)$ solution of $u_t = u_{xx}$.

Exercise 3:
The backward time, central space scheme for the diffusion equation is given by is

$$\frac{U_{j,n} - U_{j,n-1}}{\Delta t} = \frac{U_{j+1,n} - 2U_{j,n} + U_{j-1,n}}{\Delta x^{\,2}}.$$

Show that the scheme consistant. Find under what restriction on $r = \Delta t/\Delta x^2$ the scheme converges.

Exercise 4:
Using exercise 1.4, show that

$$u_x\Big|_j \Delta_x = \left[B_x + \frac{B_x^2}{2} + \frac{B_x^3}{3} + ...\right] u_j.$$