UNIVERSITY OF SURREY©

School of Engineering

Undergraduate Programmes in

Aerospace Engineering
Aerospace Materials
Chemical and Process Engineering
Civil Engineering
Engineering with Business Management
Materials Science and Engineering
Mechanical Engineering

Level 1

SE 102 Mathematics 2

Time allowed: 2 hours — Spring Semester 2002

Answer all questions. All working must be shown.
The marks for each question are shown in brackets; you should note that some questions carry more marks than others.

1. Solve the following differential equations, expressing the solution in the form $y = f(x)$:

(i) $\dfrac{dy}{dx} = -2y^2x$ [4]

(ii) $\dfrac{dy}{dx} = (y^2+1)x^2$ subject to $y(0) = 1$ [4]

(iii) $\dfrac{dy}{dx} + 3y = x$ [6]

2. Dr Gourley likes to drink a cup of coffee before his 11am lecture to the engineering students. The coffee is 200^{o}F when poured at 10.15am, and 15 minutes later it cools to 120^{o}F in a room in which the temperature is 70^{o}F. Dr Gourley will not drink the coffee until it has cooled to 90^{o}F. At what time (to the nearest minute) will he start drinking it?
[Assume that the temperature T of the coffee satisfies $dT/dt = -k(T-70)$ for some number k]. [8]

3. Solve the differential equations:

(i) $\dfrac{d^2y}{dx^2} + 7\dfrac{dy}{dx} + 10y = 0$ [4]

(ii) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 5y = x^2 - 1$ [8]

4. Calculate the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ of each of the following

(i) $z = x^3 - xy^2 + y$ [4]

(ii) $z = y\,e^{x/y}$ [4]

5. A closed rectangular box is to have a volume of $2\,\mathrm{m}^3$. The top and bottom are made from material costing 10 pence per square metre, and the sides from material costing 5 pence per square metre. Show that the cost C of the box (in pence) is given by

$$C(x,y) = 20\left(xy + \frac{1}{x} + \frac{1}{y}\right)$$

where x and y are the length and breadth of the base. Find the dimensions of the box which minimise the cost of manufacture. [You must confirm that you have a minimum]. [14]

6. For each of the following systems of simultaneous equations determine whether the system has a unique solution, an infinite number of solutions or no solution and solve where possible. [14]

$$\text{(i)}\quad \begin{aligned} x - y - 2z &= 1 \\ 2x + 3y + z &= 2 \\ 5x + 4y + 2z &= 4 \end{aligned} \qquad\qquad \text{(ii)}\quad \begin{aligned} x + y + 2z &= 8 \\ 3x + 2y + z &= 6 \\ 2x + y - z &= -3 \end{aligned}$$

7.

(i) Find the rank of the matrix

$$\begin{pmatrix} 2 & 0 & 9 & 2 \\ 1 & 4 & 6 & 0 \\ 3 & 5 & 7 & 1 \end{pmatrix}$$ [5]

(ii) Find the value of x for which the determinant

$$\begin{vmatrix} 1 & -4 & 1 \\ 8 & 3 & 3 \\ 3 & 23 & x \end{vmatrix}$$

is zero. [5]

8. Find the moment of inertia, about the x-axis, of the solid cone of mass m formed by rotating the lamina $0 \leq y \leq 2x$, $0 \leq x \leq 1$, about the x-axis. You may assume that the moment of inertia of a disk of mass M, radius a, about an axis through its centre and perpendicular to the plane of the disk, is $\frac{1}{2}Ma^2$. [8]

9. Evaluate the following double integrals:

(i) $\displaystyle\int_0^{\pi}\int_0^{x} x \sin y \, dy \, dx$ [5]

(ii) $\displaystyle\iint_D (1 + xy) \, dA$ where D is the triangle with vertices $(1, 1)$, $(2, 1)$ and $(2, 2)$. [7]

Internal Examiner: S.A. GOURLEY