UNIVERSITY OF SURREY©

School of Engineering

Undergraduate Programmes in Engineering

Level 1

SE 0102 Mathematics 2

Time allowed: 2 hours — Spring Semester 2004

Answer all questions. All working must be shown.
The marks for each question are shown in brackets; you should note that some questions carry more marks than others.

1. Solve the following differential equations, expressing the solution in the form $y = f(x)$:

(i) $\dfrac{dy}{dx} = -y\cos x$ [4]

(ii) $\dfrac{dy}{dx} = \dfrac{y^2+1}{y}$ subject to $y(1) = 2$ [5]

(iii) $x\dfrac{dy}{dx} = 4y + x^4$ [5]

2. It is noon on a fine Spring day, with air temperature 21^oC, and Inspector Poirot has just found a murder victim. He records the body temperature at 31^oC. One hour later, the temperature of the body is 29^oC. Calculate the time of death, assuming that the victim's body temperature was 37^oC at that time.
[Assume also that the victim's body temperature T satisfies $dT/dt = -k(T-21)$ for some number k]. [8]

3. Solve the differential equations:

(i) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 13y = 0$ subject to $y(0) = 1$ and $y'(0) = 0$ [6]

(ii) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - 3y = 5\sin 3x$ [7]

4. Calculate the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ of each of the following

(i) $z = x + y^2x + 4y^3x^2$ [4]

(ii) $z = x^2 \sin xy$ [4]

5. Let $z = x^3 - y^3 - 2xy + 6$.

(i) Find the first and second order partial derivatives $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}, \dfrac{\partial^2 z}{\partial x^2}, \dfrac{\partial^2 z}{\partial y^2}$ and $\dfrac{\partial^2 z}{\partial x \partial y}$. [5]

(ii) Show that z has stationary points at $(x, y) = (0, 0)$ and $(-\frac{2}{3}, \frac{2}{3})$ and determine their nature. [8]

6. Solve the system of simultaneous equations:

$$\begin{aligned} x - 2y + z &= -6 \\ 3x + y + 5z &= 7 \\ -x + 2y + 3z &= 14 \end{aligned}$$

[NB: careless errors will be heavily penalised. You should check that the values you have found fit the equations.] [7]

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7. Show that the system of simultaneous equations

$$\begin{aligned} 2x + y - z &= -3 \\ 3x + 2y + z &= 6 \\ x + y + 2z &= 9 \end{aligned}$$

is consistent but has infinitely many solutions. If $z = \alpha$, find x and y in terms of α. [7]

8.

(i) Find the rank of the matrix

$$\begin{pmatrix} 2 & 1 & 1 & 1 \\ 4 & 2 & 2 & 3 \\ 0 & 0 & 0 & 1 \\ -2 & -1 & -1 & 0 \end{pmatrix}$$ [6]

(ii) Evaluate

$$\begin{vmatrix} 5 & 1 & 8 \\ 9 & 3 & 6 \\ 12 & 4 & 2 \end{vmatrix}$$ [5]

9. A thin plate of constant density occupies the region in the first quadrant bounded by the x and y axes and the curve $y = 4 - x^2$. Sketch this region and find the coordinates of the centre of mass of the plate. [8]

10.

(i) Evaluate $\int_0^1 \int_0^x xy^3 \, dy \, dx$ [4]

(ii) Find the volume of the tetrahedron that lies in the first octant $x, y, z \geq 0$ and is bounded by the coordinate planes and the plane $z = 6 - 2x - y$. [7]

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