UNIVERSITY OF SURREY©

School of Engineering

Undergraduate Programmes in Engineering

Level 1

SE 0102 Mathematics 1b

Time allowed: 2 hours Spring Semester 2005

Answer all questions. All working must be shown.
The marks for each question are shown in brackets; you should note that some questions carry more marks than others.

1. Solve the following differential equations, expressing the solution in the form $y = f(x)$:

(i) $\dfrac{dy}{dx} = \dfrac{e^{-y}}{x}$ [3]

(ii) $\dfrac{dy}{dx} = \dfrac{\sin x}{y^2}$ subject to $y(0) = 4$ [5]

(iii) $(x+1)\dfrac{dy}{dx} + 2y = 1$ [5]

2. A thief has stolen a can of lemonade from a fridge. When he is arrested for this offence at 1.30pm, the temperature of the lemonade is 11^oC. Half an hour later the temperature of the lemonade has risen to 14^oC. The air temperature is 18^oC and the temperature in the fridge is 2^oC. Calculate, to the nearest minute, when the lemonade was stolen (giving the answer as a time of day).
[Assume that the temperature T of the lemonade satisfies $dT/dt = k(T-18)$ for some number k]. [8]

3. Solve the differential equations:

(i) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 26y = 0.$ [4]

(ii) $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 2y = 5 - 6x$ subject to $y(0) = 3$ and $y'(0) = 11$. [9]

4. Calculate the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ of each of the following

(i) $z = 3y^2 + yx + 3y^2x^4$ [4]

(ii) $z = \dfrac{2x^2}{x+y}$ [4]

5. An open-top rectangular cardboard box is to have a volume of 3 m^3. If A is the total area of cardboard and x and y are the length and breadth of the base show that

$$A = xy + \frac{6}{x} + \frac{6}{y}$$

Find the dimensions of the box which minimise the amount of cardboard required, giving each measurement to three decimal places. [NB: you must confirm that you have a minimum]. [13]

6. For each of the following systems of simultaneous equations determine whether the system has a unique solution, an infinite number of solutions or no solution and solve where possible. [14]

(i)
$$\begin{aligned} 2x + y - 3z &= -5 \\ x - y + 2z &= 12 \\ 7x - 2y + 3z &= 37 \end{aligned}$$

(ii)
$$\begin{aligned} x + y + 5z &= -2 \\ 4x + 7y + 8z &= 2 \\ 3x + 5y + 7z &= 1 \end{aligned}$$

[NB: careless errors will be heavily penalised.]

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7.

(i) Find the rank of the matrix

$$\begin{pmatrix} 5 & -2 & 5 & 6 & 1 \\ -2 & 0 & 1 & -1 & 3 \\ -1 & -2 & 8 & 3 & 10 \end{pmatrix}$$ [6]

(ii) Show that

$$\begin{vmatrix} x+1 & -1 & 2 \\ 2 & x-1 & 1 \\ 1 & -1 & x+2 \end{vmatrix} = x^3 + 2x^2$$ [6]

8. A triangular lamina is bounded by the x-axis, the line $x = 1$ and the line $y = 2x$. Sketch this lamina. The lamina rotates about the x-axis. Find its moment of inertia in terms of its total mass M. [Hint: divide the lamina up into thin vertical rods, and use the fact that the moment of inertia of a rod of length y rotating about one end is $\frac{1}{3}my^2$ where m is the rod's mass.] [7]

9. Evaluate

(i) $\int_0^1 \int_{x^2}^{x} (1 - xy)\, dy\, dx$ [5]

(ii) $\iint_D (x^2 + y^2)\, dA$ where D is the triangle with vertices $(0, 0)$, $(2, 0)$ and $(1, 1)$. [7]

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