UNIVERSITY OF SURREY©

School of Engineering

Undergraduate Programmes in Engineering

Level 1

SE 0102 Mathematics 1b

Time allowed: 2 hours | Spring Semester 2007

Answer all questions. All working must be shown.
The marks for each question are shown in brackets; you should note that some questions carry more marks than others.

1. Solve the following differential equations for y, giving the answer in the form $y = f(x)$:

(i) $\dfrac{dy}{dx} = 3x^2y^2$ subject to $y(0) = 1$, [3]

(ii) $\dfrac{dy}{dx} = (1 + y)\cos x$, [5]

(iii) $x\dfrac{dy}{dx} - 2y = x^4e^{-x}$. [5]

2. A sphere of ice is melting. Its volume changes at a rate proportional to its surface area. If V is the volume of the sphere at time t, show that V satisfies a differential equation of the form

$$\frac{dV}{dt} = -kV^{2/3}$$

where k is some positive constant. If $V = V_0$ at $t = 0$, find V as a function of t. [7]
[Volume of sphere = $\frac{4}{3}\pi r^3$, surface area of sphere = $4\pi r^2$, where r = radius]

3. Solve the following differential equations for y:

(i) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 20y = 0$ subject to $y(0) = 3$ and $y'(0) = 1$ [6]

(ii) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 5y = 3\cos x$ [7]

4. Find the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ of each of the following:

(i) $z = 1 + y + 2x + x^2y^4 + xy^3$ [4]

(ii) $z = y\sin(x^2y)$ [4]

5. Let $z = x^2 + y^3 - 6xy + 3x + 6y$.

(i) Find the first and second order partial derivatives $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}, \dfrac{\partial^2 z}{\partial x^2}, \dfrac{\partial^2 z}{\partial y^2}$ and $\dfrac{\partial^2 z}{\partial x \partial y}$. [5]

(ii) Show that z has stationary points at $(x, y) = (\frac{27}{2}, 5)$ and $(\frac{3}{2}, 1)$ and determine their nature. [8]

6. Write the system of simultaneous equations

$$\begin{aligned} x + y - z &= -6 \\ 3x - 2y + 4z &= 5 \\ -2x - y + 7z &= 33 \end{aligned}$$

in matrix form [2 marks], and hence solve the system [2 marks for each correct value, provided you have shown the working]. [8]

[SEE NEXT PAGE

7. For a system of simultaneous equations of the form $A\mathbf{x} = \mathbf{b}$, give a condition involving the ranks of certain matrices for the system to be consistent.
Find a condition on the numbers a, b and c such that the simultaneous equations

$$\begin{aligned} x - 2y + 5z &= a \\ 4x - 5y + 8z &= b \\ -3x + 3y - 3z &= c \end{aligned}$$

are consistent. (You are *not* required to actually solve them). [6]

8. (i) Find the rank of the matrix

$$\begin{pmatrix} -1 & 0 & -1 & -2 & -1 \\ 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{pmatrix}$$ [6]

(ii) Find the two values of x such that

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & 1 \\ 1 & -3 & 9 \end{vmatrix} = 0$$ [6]

9. Evaluate the following double integrals:

(i) $\displaystyle\int_1^2 \int_0^{x-1} y\, dy\, dx$ [5]

(ii) $\displaystyle\iint_D (x^2 - xy)\, dA$ where D is the region enclosed between the curves $y = x$ and $y = 3x - x^2$. [Hint: integrate in the y direction first]. Your solution should include a clear sketch of the region D. [8]

10. A thin plate of constant density occupies the region bounded by the x-axis, the y-axis and the line $y = 4 - 2x$. Give a clear sketch of the plate.
If the plate rotates about the x-axis, find its moment of inertia in terms of its total mass M. [Hint: divide the plate up into thin vertical rods, and use the fact that the moment of inertia of a rod of length y rotating about an axis perpendicular to the rod and passing through one end, is $\frac{1}{3}my^2$ where m is the rod's mass.] [7]

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