UNIVERSITY OF SURREY©

Faculty of Engineering and Physical Sciences

Undergraduate Programmes in Engineering

Level 1

ENG1002 Mathematics 1b

Time allowed: 2 hours — Spring Semester 2009

Answer all questions. All working must be shown. Approved calculators may be used. *The marks for each question are shown in brackets; you should note that some questions carry more marks than others.*

1. Solve the following differential equations for y, giving the answer in the form $y = f(x)$:

(i) $\dfrac{dy}{dx} = 2e^{2x-y}$ subject to $y(0) = 1$, [5]

(ii) $\dfrac{dy}{dx} = (2y+1)\cos 3x$, [4]

(iii) $x\dfrac{dy}{dx} = -3y + 6x$. [5]

2. A body moving in a particular kind of medium experiences a resistance such that its velocity v satisfies $dv/dt = -k\,v^{3/2}$ where k is a positive constant. Find an expression for v, given that at time $t = 0$ the velocity is v_0.
The displacement x of the body satisfies $dx/dt = v$. If $x = 0$ when $t = 0$ show that, for $t > 0$,

$$x = \frac{2}{k}\sqrt{v_0}\left(1 - \frac{2}{kt\sqrt{v_0} + 2}\right)$$ [6]

3. Solve the following differential equations for y:

(i) $\dfrac{d^2y}{dx^2} + 16y = 0$ subject to $y(0) = 3$ and $y'(0) = -2$ [6]

(ii) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - 15y = 2e^{4x}$ [7]

4. Find the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ of each of the following:

(i) $z = 2x^4 + 3xy^5 - 3x^2y^3 + 8y$ [4]

(ii) $z = y\cos(x^2 + y)$ [4]

5. Let $z = -x^3 - y^3 + 6x^2 + 6xy - 12x - 12y + 1$.

(i) Find the first and second order partial derivatives $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}, \dfrac{\partial^2 z}{\partial x^2}, \dfrac{\partial^2 z}{\partial y^2}$ and $\dfrac{\partial^2 z}{\partial x \partial y}$. [5]

(ii) Show that z has stationary points at $(x, y) = (2, 0)$ and $(4, 2)$ and determine their nature. [7]

6. Write the system of simultaneous equations

$$\begin{aligned} -4x + 2y + 3z &= 6 \\ x + y + 4z &= 4 \\ -9x + 3y + 4z &= 10 \end{aligned}$$

in matrix form [2 marks]. Solve the system and check that the values you have found fit all three equations. [2 marks for each correct value, provided you have shown the working]. [8]

[SEE NEXT PAGE

7. Find the value of k for which the simultaneous equations

$$\begin{aligned} x + 5y + 3z &= 0 \\ 7x + 5y + kz &= 0 \\ x + 2y + kz &= 0 \end{aligned}$$

have solutions **other** than $(x, y, z) = (0, 0, 0)$. [You are **not** required to actually find the other solutions.] [7]

8. (i) Find the rank of the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$ [5]

(ii) Show that

$$\begin{vmatrix} 4-\lambda & 2 & 2 \\ 2 & 4-\lambda & 2 \\ 2 & 2 & 4-\lambda \end{vmatrix} = (\lambda - 2)^2(8 - \lambda)$$ [7]

9. Evaluate the following double integrals:

(i) $\displaystyle\int_2^4 \int_0^{x-2} y^2 \, dy \, dx$ [5]

(ii) $\displaystyle\iint_D e^{y/x} \, dA$ where D is the region enclosed between the curve $y = x^3$, the x-axis and the line $x = 1$. [Hint: integrate in the y direction first]. Your solution should include a clear sketch of the region D. [8]

10. Find the moment of inertia, about the x-axis, of the solid object formed by rotating the lamina $0 \le y \le \sin x$, $0 \le x \le \pi$, about the x-axis. You may leave your answer in terms of the density ρ (the mass per unit volume) of the object.
[You may assume that the moment of inertia of a disk of mass M, radius a, about an axis through its centre and perpendicular to the plane of the disk, is $\frac{1}{2}Ma^2$. You may also assume that $\displaystyle\int_0^{\pi} \sin^4 x \, dx = \frac{3\pi}{8}$]. [7]

Internal Examiner: S.A. GOURLEY