

## Mathematics 1b: Sheet 3 (will not be marked)

1. Find the position of the centre of mass of

(i) the triangular lamina formed by the line  $y = 1 - x$  and the  $x$  and  $y$  axes;

(ii) the semicircular lamina  $x^2 + y^2 = a^2$ ,  $y \geq 0$ .

2. A lamina is bounded by the curve  $y^2 = 4x$ , the  $x$ -axis and the line  $x = 4$ . Sketch this lamina. Calculate the moment of inertia of the lamina about the  $x$ -axis. (Hint: Divide the lamina into a large number of thin vertical 'rods'. In lectures we have calculated the moment of inertia of a rod rotating about one end).

3. Evaluate the following double integrals:

$$\int_0^1 \int_{-1}^1 (x + y + 1) dx dy, \quad \int_0^{\ln 3} \int_0^{\ln 2} e^{x+y} dx dy, \quad \int_0^2 \int_0^{9-4x^2} 3x dy dx$$

**Answers:**

1. (i)  $(\frac{1}{3}, \frac{1}{3})$ , (ii)  $(0, \frac{4a}{3\pi})$ .

2.  $16m/5$  where  $m$  is the lamina's mass.

3. 3, 2, 6