Mathematics 1b: Sheet 4 (assessed)

Issued 2nd March 2012. Hand in on Monday 12 March at Teaching Support Office. Please state on the cover sheet which tutorial group you are in.

1. Evaluate the following double integrals:

(i) $\int_0^2 \int_1^4 y\sqrt{x}\, dy\, dx$. [3]

(ii) $\int_\pi^{2\pi} \int_0^\pi (\sin x + \cos y)\, dx\, dy$. [3]

(iii) $\iint_D (1 - x + 2y)\, dA$ where D is the region enclosed between $x + y = 1$ and $x^2 + y = 1$. [5]

(iv) $\iint_D e^{x^2}\, dA$ where D is the triangle formed by the x-axis, the line $3y = x$ and the line $x = 3$ [hint: integrate in the y direction first]. [5]

2. Draw the plane $\frac{1}{3}x + \frac{1}{4}y + \frac{1}{5}z = 1$ showing where it intersects the x, y and z axes. Calculate the volume of the tetrahedron enclosed by this plane and the coordinate planes. [6]

3. Calculation of moments of inertia sometimes requires the evaluation of double integrals. An example is a thin rectangular metal plate of length a and width b spinning about an axis which passes through one corner and is perpendicular to the plate. Taking the plate to occupy the region $D = \{(x, y) : \ 0 \le x \le a,\ 0 \le y \le b\}$, the formula $I = \sum r^2\, \delta m$ taught in lectures becomes

$$I = \sigma \iint_D (x^2 + y^2)\, dx\, dy$$

where σ is the mass per unit area. If m is the mass of the plate show that

$$I = \frac{1}{3} m(a^2 + b^2).$$ [5]

Maximum mark 27