Mathematics 1b: Sheet 6 (will not be marked)

1. Solve the following differential equations, giving the final answer in the form $y = f(x)$ where possible

(a) $x + \sin y \dfrac{dy}{dx} = 0$ subject to $y(0) = \pi/4$

(b) $\dfrac{dy}{dx} = xe^{y-x}$

(c) $\dfrac{dy}{dx} = \dfrac{xy - y}{y+1}$ subject to $y(2) = 1$

2. Solve the following differential equations for y

(a) $\dfrac{dy}{dx} + y = 5e^x$

(b) $\dfrac{dy}{dx} + \frac{1}{x}y = x^3 - 1$ subject to $y(1) = 2$

(c) $\dfrac{dy}{dx} + \dfrac{2}{x+1}y = 1$

(d) $t\dfrac{dy}{dt} + 2y = e^t$

3. If glucose is fed intravenously at a constant rate, the change in the overall concentration $c(t)$ of glucose in the blood can be described by the differential equation

$$\frac{dc}{dt} = \frac{G}{100V} - kc$$

where G, V and k are positive constants. Find an expression for $c(t)$, using c_0 to denote $c(0)$. What does $c(t)$ settle down to, as $t \to \infty$?

Answers:

1. (a) $y = \cos^{-1}\left(\dfrac{x^2}{2} + \dfrac{1}{\sqrt{2}}\right)$.
(b) $y = -\ln(x\,e^{-x} + e^{-x} - A)$,
(c) $y + \ln y = \frac{1}{2}x^2 - x + 1$.

2. (a) $y = \frac{5}{2}e^x + c\,e^{-x}$
(b) $y = \frac{x^4}{5} - \frac{1}{2}x + \frac{23}{10x}$
(c) $y = \frac{x+1}{3} + \frac{c}{(x+1)^2}$
(d) $y = \frac{e^t}{t} - \frac{e^t}{t^2} + \frac{c}{t^2}$.

3. $c(t) = \dfrac{G}{100Vk} + \left(c_0 - \dfrac{G}{100Vk}\right)e^{-kt}$.