Mathematics 1b: Sheet 7 (will not be marked)

1. Solve the differential equations

(a) $y'' - 2y' + 17y = 0$

(b) $y'' + 4y' + 3y = 0$

(c) $y'' + 2y' = 0$ subject to $y(0) = 3$ and $y'(0) = -2$

2. Solve the differential equations

(a) $y'' - 6y' + 9y = x$

(b) $y'' - 4y' + 8y = e^{5x}$

(c) $y'' + 2y' + 2y = \sin 3x$

(d) $y'' + 6y' + 8y = 3e^{-2x}$ subject to $y(0) = 1$ and $y'(0) = -3$

3. The charge q on a capacitor in a certain electrical circuit satisfies

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E$$

where L, R, C and E are constants. Show that if $2L = CR^2$ the general solution of this is

$$q = e^{-t/(CR)}\left(A\cos\frac{t}{CR} + B\sin\frac{t}{CR}\right) + CE$$

Answers:

1. (a) $y = e^x(A\cos 4x + B\sin 4x)$
(b) $y = Ae^{-3x} + Be^{-x}$
(c) $y = 2 + e^{-2x}$
2. (a) $y = (Ax + B)e^{3x} + \frac{1}{9}x + \frac{2}{27}$
(b) $y = e^{2x}(A\cos 2x + B\sin 2x) + \frac{1}{13}e^{5x}$
(c) $y = e^{-x}(A\cos x + B\sin x) - \frac{6}{85}\cos 3x - \frac{7}{85}\sin 3x$
(d) $y = \frac{5}{4}e^{-4x} - \frac{1}{4}e^{-2x} + \frac{3}{2}x\,e^{-2x}$