Mathematics 1b: Sheet 8 (will not be marked)

1. Evaluate $A+B$, $A-B$, $4A$ and $4A+B$ for each of the following cases.

(i) $$A = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 4 & 0 & 2 & 1 \\ 2 & -5 & 1 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 3 & -4 & 1 & 2 \\ 1 & 5 & 0 & 3 \\ 2 & -2 & 3 & -1 \end{pmatrix}$$

(ii) $$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, \qquad B = \begin{pmatrix} -3 & -2 \\ 1 & -5 \\ 4 & 3 \end{pmatrix}$$

2. Find, where possible, the matrix products AB and BA in each of the following cases

(i) $$A = \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix}, \qquad B = \begin{pmatrix} 6 & 8 \\ 1 & -4 \end{pmatrix}$$

(ii) $$A = \begin{pmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix}$$

(iii) $$A = \begin{pmatrix} -4 & 6 & 2 \\ -2 & -2 & 3 \\ 1 & 1 & 8 \end{pmatrix}, \qquad B = \begin{pmatrix} -2 & 4 & 7 & 6 \\ -3 & 1 & 2 & 4 \\ 0 & 1 & 6 & -2 \end{pmatrix}$$

3. Find AB if

$$A = \begin{pmatrix} 4 & 5 & 6 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$$

4. Find A^2 and A^3 when

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$$

5. Find the ranks of the following matrices by reducing them to row echelon form:

(i) $$\begin{pmatrix} 1 & 2 & 4 \\ -2 & 0 & 1 \\ -1 & 2 & 5 \end{pmatrix}$$ (ii) $$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{pmatrix}$$ (iii) $$\begin{pmatrix} 2 & 4 & 3 & 4 \\ 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 3 & 7 & 4 & 6 \end{pmatrix}$$

(iv) $$\begin{pmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{pmatrix}$$ (v) $$\begin{pmatrix} 0 & -1 & 2 \\ 2 & 0 & 2 \\ 3 & 4 & 7 \end{pmatrix}$$

6. Solve the following system of equations by reducing the corresponding augmented matrix to row echelon form.

$$\begin{aligned} x_1 - 5x_2 + 3x_3 &= -1 \\ 4x_1 - 18x_2 + 15x_3 &= 4 \\ -2x_1 + 11x_2 - 11x_3 &= -7 \end{aligned}$$

7. Find the value of λ for which the equations

$$\begin{aligned} x_1 - 2x_2 + 9x_3 &= 4 \\ 2x_1 - 6x_2 + 21x_3 &= 9 \\ 3x_1 + 10x_3 &= 6 \\ -x_1 + 4x_2 - 13x_3 &= \lambda \end{aligned}$$

are consistent, and solve them with this value of λ. (N.B. Take care when you write out the augmented matrix; note the third equation has no x_2 term).

Answers on back

Answers:

1. (i) $A+B=\begin{pmatrix} 4 & -2 & 0 & 2 \\ 5 & 5 & 2 & 4 \\ 4 & -7 & 4 & 1 \end{pmatrix}$, $\qquad A-B=\begin{pmatrix} -2 & 6 & -2 & -2 \\ 3 & -5 & 2 & -2 \\ 0 & -3 & -2 & 3 \end{pmatrix}$

$4A=\begin{pmatrix} 4 & 8 & -4 & 0 \\ 16 & 0 & 8 & 4 \\ 8 & -20 & 4 & 8 \end{pmatrix}$, $\qquad 4A+B=\begin{pmatrix} 7 & 4 & -3 & 2 \\ 17 & 5 & 8 & 7 \\ 10 & -22 & 7 & 7 \end{pmatrix}$

(ii) $A+B=\begin{pmatrix} -2 & 0 \\ 4 & -1 \\ 9 & 9 \end{pmatrix}$, $\qquad A-B=\begin{pmatrix} 4 & 4 \\ 2 & 9 \\ 1 & 3 \end{pmatrix}$

$4A=\begin{pmatrix} 4 & 8 \\ 12 & 16 \\ 20 & 24 \end{pmatrix}$, $\qquad 4A+B=\begin{pmatrix} 1 & 6 \\ 13 & 11 \\ 24 & 27 \end{pmatrix}$

2. (i) $AB=\begin{pmatrix} 16 & 0 \\ 17 & 28 \end{pmatrix}$, $\qquad BA=\begin{pmatrix} 36 & 16 \\ -10 & 8 \end{pmatrix}$

(ii) $AB=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\qquad BA=\begin{pmatrix} -11 & 6 & -1 \\ -22 & 12 & -2 \\ -11 & 6 & -1 \end{pmatrix}$

(iii) $AB=\begin{pmatrix} -10 & -8 & -4 & -4 \\ 10 & -7 & 0 & -26 \\ -5 & 13 & 57 & -6 \end{pmatrix}$, $\qquad BA$: impossible

3. $AB=(17)$

4. $A^2=\begin{pmatrix} 5 & -3 & 1 \\ 2 & 1 & 4 \\ 3 & -1 & 2 \end{pmatrix}$, $\qquad A^3=\begin{pmatrix} 11 & -8 & 0 \\ 8 & -1 & 8 \\ 8 & -4 & 3 \end{pmatrix}$

5. (i) 2 (ii) 2 (iii) 3 (iv) 3 (v) 3

6. $x_1=-2$, $x_2=1$, $x_3=2$.

7. $\lambda=-\frac{43}{8}$. $x_1=\frac{3}{4}$, $x_2=\frac{1}{16}$, $x_3=\frac{3}{8}$.