UNIVERSITY OF SURREY©

School of Electronics and Physical Sciences

Undergraduate Programmes in Electronic Engineering

Level 1

Module EE1.MAB Mathematics B

Time allowed: 2 hours | Spring Semester 2007

Answer all questions. All working must be shown.

The marks for each question are shown in brackets. You should note that some questions carry more marks than others.

You may use the **Tables of Constants, Formulae and Transforms**. Approved calculators may be used.

1.

(i) Find the binomial expansion of $(1 - x^2)^{-3}$ up to and including the term in x^6. [5]

(ii) Use the expansion of $\ln(1 + x)$ (which you may quote from the booklet) to find the Maclaurin expansions of $\ln(1 + 2x)$ and $\ln((1 + 2x)(1 + x))$, in each case up to and including the term in x^3. [5]

2. Using L'Hopital's rule or otherwise, find the limits

(i) $\lim_{x \to 2} \dfrac{x - 2}{x^3 - 8}$ (ii) $\lim_{x \to 1} \dfrac{2x^2 - (3x + 1)\sqrt{x} + 2}{x - 1}$ (iii) $\lim_{x \to 0} \dfrac{1 - \cos 3x}{x^2}$ $[3, 4, 5]$

3.

(i) Simplify the following expressions as far as possible: (a) $\sinh(2 \ln x)$, (b) $\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x)$ [3,3]

(ii) Using an appropriate substitution, find

$$\int_0^{1/3} \frac{6\, dx}{\sqrt{1 + 9x^2}}$$

[6]

4. Solve the following differential equations, expressing the solution in the form $y = f(x)$:

(i) $\dfrac{dy}{dx} = yx^3$ [4]

(ii) $e^{2x} \dfrac{dy}{dx} = y^2$ subject to $y(0) = 1$ [6]

(iii) $(x + 1)\dfrac{dy}{dx} + 2y = 1$ [6]

5. Solve the differential equations:

(i) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 5y = 0$ subject to $y(0) = -2$ and $y'(0) = 1$ [7]

(ii) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 2y = \cos 2x$ [8]

6. Show that the Newton Raphson method, applied to the equation $x^6 - 2x^2 - 1 = 0$, leads to the recursion formula

$$x_{n+1} = x_n - \frac{x_n^6 - 2x_n^2 - 1}{6x_n^5 - 4x_n}$$

Explain why the equation must have a root between 1 and 2. Find an approximation to this root accurate to three decimal places. [10]

7.

(i) Evaluate $\int_0^2 \int_0^{4-y^2} y \, dx \, dy$. [5]

(ii) By converting to polar coordinates r and θ defined by $x = r\cos\theta$, $y = r\sin\theta$, evaluate $\iint_D e^{-(x^2+y^2)} \, dA$ where D is the portion of the circle centred at the origin and of radius 1 that lies in the first quadrant.
[You may quote that the Jacobian of the transformation is r]. [8]

8. A function $f(t)$ is 2π periodic and is given for $-\pi < t < \pi$ by $f(t) = t$.

(i) Draw the graph of $f(t)$ for $-3\pi < t < 3\pi$. [4]

(ii) How do we know that the Fourier series of $f(t)$ will contain only sine terms? [2]

(iii) Find the Fourier series of $f(t)$ in the form

$$f(t) = \sum_{n=1}^{\infty} b_n \sin nt$$

by calculating the value of b_n for $n = 1, 2, 3, \ldots$. [6]

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