UNIVERSITY OF SURREY©

Faculty of Engineering and Physical Sciences

Undergraduate Programmes in Electronic Engineering

Level 1

Module EE1.MAB Mathematics B

Time allowed: 2 hours

Spring Semester 2008

Answer all questions. All working must be shown.

The marks for each question are shown in brackets. You should note that some questions carry more marks than others.

You may use the **Tables of Constants, Formulae and Transforms**. Approved calculators may be used.

1.

(i) Find the binomial expansion of $(1-2x)^{-4}$ up to and including the term in x^4. [5]

(ii) Integrals like $\int \sin(x^2)\,dx$ arise in the diffraction of light but they cannot be evaluated exactly. Find the Maclaurin series for $\sin(x^2)$ up to and including the term in x^{10}, and use it to find an approximate value for

$$\int_0^1 \sin(x^2)\,dx$$

giving your answer to four decimal places.

[5]

2. Using L'Hopital's rule or otherwise, find the limits

$$\text{(i)}\ \lim_{x\to 2}\frac{x^4-16}{x^2-4} \qquad \text{(ii)}\ \lim_{x\to 0}\frac{\tan 3x}{\tan 5x} \qquad \text{(iii)}\ \lim_{x\to 1}\frac{x^3-1}{\sqrt{x}-1}$$ [3, 4, 4]

3.

(i) Prove that

$$\tanh(\ln x) = \frac{x^2-1}{x^2+1}$$ [4]

(ii) Find

$$\int \sinh^2 2x\,dx$$ [5]

and

$$\int \frac{dx}{\sqrt{x^2-6x-7}}$$ [5]

4. Solve the following differential equations, expressing the solution in the form $y = f(x)$:

(i) $\dfrac{dy}{dx} + \dfrac{x}{y} = 0$ [4]

(ii) $\dfrac{dy}{dx} = \dfrac{x+3x^2}{y^2}$ subject to $y(0) = 6$ [6]

(iii) $\dfrac{dy}{dx} + \dfrac{2y}{x} = \dfrac{\sin 3x}{x^2}$ [6]

[SEE NEXT PAGE

5. Solve the differential equations:

(i) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 20y = 0 \quad$ subject to $y(0) = 1$ and $y'(0) = -2$ [7]

(ii) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 13y = 3e^{-2x}$ [8]

6. Sketch the graphs of the left and right hand sides of the equation $e^{2x} = 3 - x^2$, and show that the equation must have a root between 0 and $\sqrt{3}$. Show that the application of the Newton Raphson method to this equation leads to the recursion formula

$$x_{n+1} = x_n - \left(\frac{e^{2x_n} + x_n^2 - 3}{2e^{2x_n} + 2x_n} \right)$$

Find an approximation to the root accurate to four decimal places. [10]

7.

(i) Evaluate $\displaystyle\int_0^1 \int_0^{2-2x} (x + 3x^2 + xy)\, dy\, dx.$ [7]

(ii) By converting to polar coordinates r and θ defined by $x = r\cos\theta$, $y = r\sin\theta$, evaluate

$$\int_0^\infty \int_0^\infty \frac{1}{(1 + x^2 + y^2)^2}\, dx\, dy.$$

[You may quote that the Jacobian of the transformation is r]. [7]

8. A function $f(t)$ is 2π periodic and is given for $-\pi < t < \pi$ by $f(t) = t^2$.

(i) Draw the graph of $f(t)$ for $-3\pi < t < 3\pi$. [2]

(ii) Show that the Fourier series of $f(t)$ is

$$\frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos nt.$$

[8]

Internal Examiner: S.A. GOURLEY