UNIVERSITY OF SURREY©

Faculty of Engineering and Physical Sciences

Undergraduate Programmes in Electronic Engineering

Level 1

Module EEE1018: Mathematics

Time allowed: 2 hours

Spring Semester 2010

Answer all questions. All working must be shown.

The marks for each question are shown in brackets. You should note that some questions carry more marks than others.

You may use the **Tables of Constants, Formulae and Transforms**. Approved calculators may be used.

1.

(i) Find the binomial expansion of $\dfrac{1}{(1-2x)^5}$ up to and including the term in x^3. [5]

(ii) Use the expansion of e^x (which you may quote from the booklet) to find the Maclaurin expansion of e^{-2x^2} up to and including the term in x^6. Deduce that, for small x,

$$\frac{e^{-2x^2}-1}{x^2} \approx -2 + 2x^2 - \tfrac{4}{3}x^4 + \cdots$$ [5]

and hence find an approximate value for

$$\int_0^{\frac{1}{5}} \frac{e^{-2x^2}-1}{x^2}\,dx$$

expressing the answer to 3 decimal places. [4]

2. Using L'Hopital's rule or otherwise, find the limits

(i) $\displaystyle\lim_{x\to 0}\frac{\sin \pi x}{\sin x}$ (ii) $\displaystyle\lim_{x\to 0}\frac{e^x-1-x}{x^2}$ (iii) $\displaystyle\lim_{x\to\infty} x\tan(2/x)$ [hint: let $y = 1/x$]

[3, 4, 4]

3.

(i) If $y = \tanh^{-1}x$ show that $e^y - e^{-y} = x(e^y + e^{-y})$ and, by solving this equation for y, deduce that

$$\tanh^{-1}x = \tfrac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$ [5]

and hence that

$$\tanh^{-1}\left(\frac{e^2-1}{e^2+1}\right) = 1.$$ [3]

(ii) Using an appropriate substitution, find

$$\int \frac{dx}{\sqrt{x^2-4x+20}}$$ [5]

in terms of an inverse hyperbolic function.

4. Solve the following differential equations, expressing the solution in the form $y = f(x)$:

(i) $\dfrac{dy}{dx} = y\cos 5x$ [4]

(ii) $\dfrac{dy}{dx} = (y^2+1)(x+1)$ subject to $y(0) = 1$ [6]

(iii) $x\dfrac{dy}{dx} + 2y = e^x$ [5]

[SEE NEXT PAGE]

5. Solve the differential equations:

(i) $\dfrac{d^2y}{dx^2} + 10\dfrac{dy}{dx} + 29y = 0$ subject to $y(0) = 1$ and $y'(0) = 2$. [7]

(ii) $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} - 18y = 2e^{4x}$. [7]

6. Sketch the graphs of the left and right hand sides of the equation $\ln x = 2 - x^2$ and show that the equation must have a root between 1.2 and 1.4. Show that the application of the Newton Raphson method to this equation leads to the recursion formula

$$x_{n+1} = x_n - \left(\frac{x_n^2 + \ln x_n - 2}{2x_n + 1/x_n}\right).$$

Find an approximation to the root accurate to four decimal places. [10]

7.

(i) Evaluate $\displaystyle\int_0^1 \int_0^{2x} (4 + y - 3x)\, dy\, dx$. [6]

(ii) By converting to polar coordinates r and θ defined by $x = r\cos\theta$, $y = r\sin\theta$, evaluate

$$\iint_D (x^3 + xy^2)\, dA$$

where D is the part of the circle $x^2 + y^2 \leq 9$ that lies to the right of the y-axis. [You may quote that the Jacobian of the transformation is r]. [7]

8. The complex form of the Fourier series of a T-periodic function $f(t)$ is

$$\sum_{n=-\infty}^{\infty} c_n\, e^{2jn\pi t/T}$$

where

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-2jn\pi t/T}\, dt \qquad \text{for } n = 0, \pm1, \pm2, \pm3, \ldots.$$

If $f(t)$ is the periodic function such that $f(t) = t$ for $0 < t < 2\pi$ and $f(t + 2\pi) = f(t)$ for all t, show that $c_0 = \pi$ and find c_n for $n = \pm1, \pm2, \ldots$. Show also that

$$\sum_{n=-1}^{1} c_n\, e^{jnt} = \pi - 2\sin t.$$

[10]

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