Calculus (Spring): Sheet 3

1. Find the maximum value of $f(x, y, z) = 3x - 2y + z$ subject to the constraint $x^2 + y^2 + z^2 = 14$. *Answer: 14.*

2. Find the volume of the largest rectangular box that can be fitted inside the ellipsoid

$$4x^2 + 9y^2 + 36z^2 = 36$$

with all eight of its vertices touching the ellipsoid, if the edges of the box are parallel to the coordinate axes. *Answer:* $16\sqrt{3}/3$

3. Find the equation of the tangent plane to the surface $z = 4x^3y^2 + 2y$ at the point $(1, -2, 12)$. *Answer:* $48x - 14y - z = 64$.

4. Evaluate the following double integrals:

$$\int_0^1 \int_{-1}^1 (x+y+1)\, dx\, dy, \qquad \int_0^{\ln 3} \int_0^{\ln 2} e^{x+y}\, dx\, dy, \qquad \int_0^2 \int_0^{9-4x^2} 3x\, dy\, dx \qquad \textit{Answers}: 3,\ 2,\ 6$$

5. Show that

$$\int_0^1 \int_{y^2}^y \sqrt{xy}\, dx\, dy = \frac{2}{27}, \qquad \int_1^\infty \int_{e^{-x}}^1 \frac{1}{x^3 y}\, dy\, dx = 1.$$

6. Evaluate the following double integrals:

(i) $\iint_D (x + y)\, dA$ where D is the region between $y = x^3$ and $y = x^4$ for $0 \leq x \leq 1$

(ii) $\iint_D e^{x^2}\, dA$ where D is the triangle formed by the x-axis, the line $2y = x$ and the line $x = 2$. *Answer:* $\frac{1}{4}(e^4 - 1)$.

7. Show that the volume of the tetrahedron in the first octant bounded by the coordinate planes and the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, \qquad a, b, c > 0$$

is $\frac{1}{6}abc$.

Please hand your work in at the lecture on Wednesday 5th May.

The lecture on Monday 3rd May will be used as a tutorial.