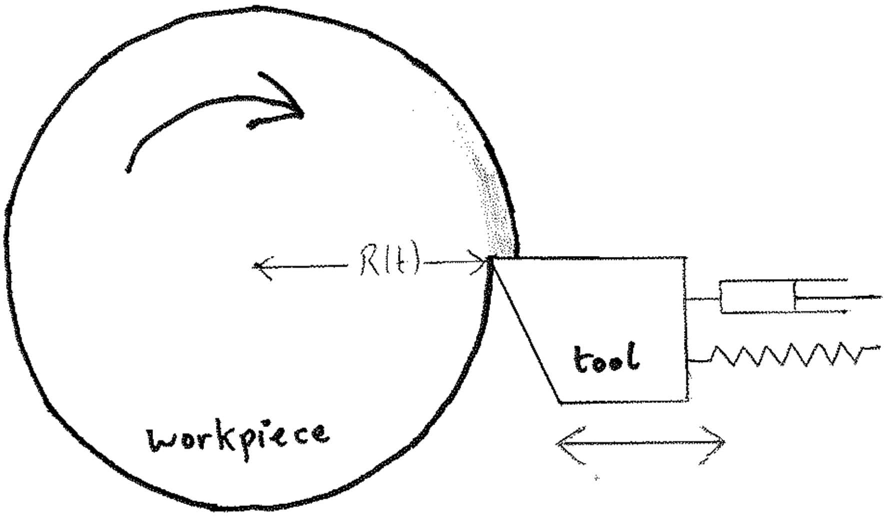
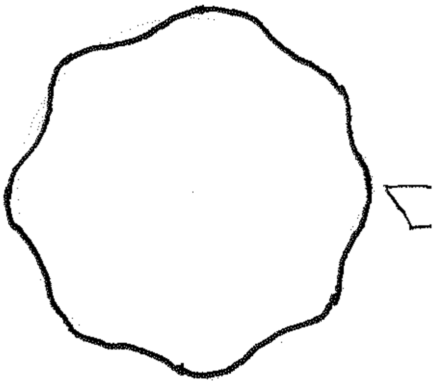
Machining vibrations (chatter)





Angular velocity $= \Omega$ radians/sec.

time for one revolution $= \dfrac{2\pi}{\Omega}$

Suppose d_0 = desired steady state chip thickness

Need to move tool inwards such that

$$R(t) = R_0 - \frac{d_0 \Omega}{2\pi} t$$

(R_0: initial radius)

But the tool may vibrate. Suppose that, in fact,

$$R(t) = R_0 - \frac{d_0 \Omega t}{2\pi} - y(t)$$

Then the chip thickness $d(t)$ at time t is

$$d(t) = \left[R_0 - \frac{d_0 \Omega}{2\pi}\left(t - \frac{2\pi}{\Omega}\right) - y\left(t - \frac{2\pi}{\Omega}\right)\right] - \left[R_0 - \frac{d_0 \Omega t}{2\pi} - y(t)\right]$$

$$\boxed{d(t) = d_0 + y(t) - y(t - \tau)} \qquad \boxed{\tau = \frac{2\pi}{\Omega}}$$

Consider forces on tool:

$$m\frac{d^2y}{dt^2} = \underbrace{(\text{force from spring})}_{\text{Hooke's law}} - \underbrace{(\text{cutting force})}_{\text{depends on chip thickness } d(t)}$$

In <u>steady state</u>, with $y=0$,

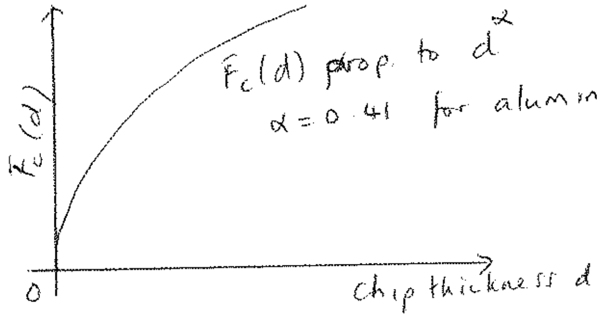
Cutting force on tool = force from spring

$$\underbrace{F_c(d_0)}_{\text{cutting force, depends on chip thickness}} = k\, l_0$$

k : spring constant

l_0 : compression in equilibrium

Experiments suggest:



In <u>vibration</u>:

$$m\frac{d^2y}{dt^2} = \underbrace{k(l_0 - y(t))}_{\text{force from spring}} - \underbrace{F_c(d(t))}_{\text{cutting force}}$$

$$\simeq k(l_0 - y(t)) - \left(F_c(d_0) + (d(t) - d_0)F_c'(d_0)\right)$$

$$m\frac{d^2y}{dt^2} + k\,y(t) = -\left(y(t) - y(t-\tau)\right)F_c'(d_0)$$

Including a damping term:

$$\boxed{m\frac{d^2y}{dt^2} + \zeta\frac{dy}{dt} + k y(t) + \left(y(t) - y(t-\tau)\right)F_c'(d_0) = 0}$$

$$y(t) = e^{\lambda t} \quad \Rightarrow$$

$$m\lambda^2 + \zeta\lambda + k + F_c'(d_0)\left(1 - e^{-\lambda\tau}\right) = 0$$

$$\boxed{\begin{array}{l}\underline{\text{Thm}}: \text{ if } \Omega\zeta > 2\pi F_c'(d_0) \text{ then all roots of the characteristic eqn satisfy } \operatorname{Re}\lambda < 0, \text{ so that} \\ \lim_{t\to\infty} y(t) = 0.\end{array}}$$

<u>Proof</u> note $\Omega\zeta > 2\pi F_c'(d_0) \iff \zeta > F_c'(d_0)\tau$

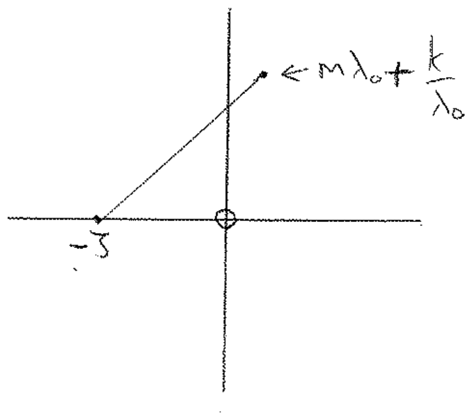
Suppose $\exists$ a root λ_0 with $\operatorname{Re}\lambda_0 \geq 0$.

$$\left|m\lambda_0^2 + \zeta\lambda_0 + k\right| = F_c'(d_0)\left|1 - e^{-\lambda_0\tau}\right|$$

$$= F_c'(d_0)\left|\lambda_0\int_0^\tau e^{-\lambda_0 s}\,ds\right|$$

$$\leq F_c'(d_0)\,|\lambda_0|\,\tau \qquad \text{since } \operatorname{Re}\lambda_0 \geq 0.$$

$$\therefore \underbrace{\left|m\lambda_0 + \frac{k}{\lambda_0} + \zeta\right|}_{\substack{\text{distance between} \\ m\lambda_0 + k/\lambda_0 \text{ and } -\zeta}} \leq F_c'(d_0)\tau$$



But $\operatorname{Re}\lambda_0 \geq 0 \iff \operatorname{Re}\, m\lambda_0 + \dfrac{k}{\lambda_0} \geq 0$

$\therefore$ distance is always $\geq \zeta$

$\therefore \quad \zeta \leq F_c'(d_0)\tau$ contradiction.

$\therefore$ to avoid chatter: can speed up rotation (increase Ω) or increase damping (increase ζ).