

Mathematics B (Elec Eng): revision exercises 2

1. Solve the differential equations

(i) $\frac{dy}{dx} = e^{2x} y$

(ii) $\frac{dy}{dx} = (xy)^3$

(iii) $\frac{dy}{dx} = x e^{y-x}$

(iv) $\frac{dy}{dx} = (1 + y^2)e^x$

(v) $\frac{dy}{dx} - 3y = x e^{3x}$ subject to $y(0) = 4$

(vi) $x \frac{dy}{dx} - 2y = 2x^2 + x$

2. Solve the differential equations

(i) $y'' + 2y' + 5y = 2x^2 + x$

(ii) $y'' - 2y' - 3y = 7e^{2x}$

(iii) $y'' + 10y' + 25y = 2 \cos 2x$

(NB: for the exam you must memorize the method including all the various cases).

3. Use the Newton-Raphson method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

(NB: you must memorize this formula for your exam) to solve the equations

(i) $x^3 + 6x^2 + 9x + 1 = 0$ (take as initial guess $x_0 = 0$)

(ii) $7x^3 + x - 5 = 0$

(iii) $2x - \sin x = 1$

4. Evaluate the double integrals:

$$(i) \int_0^2 \int_0^1 (4 - x^2 - y) dx dy \quad (ii) \int_1^3 \int_x^{x^2} (x + 2y) dy dx$$

5. Evaluate the double integrals:

(i) $\iint_D (x + y) dA$, where D is the triangle with vertices $(0, 0)$, $(0, 4)$ and $(1, 4)$.

(ii) $\iint_D xy^2 dA$, where D is the region in the first quadrant bounded by the curves $y = x^2$ and $x = y^2$.

Answers:

1. (i) $y = A \exp(\frac{1}{2} \exp(2x))$, (ii) $y = \sqrt{\frac{2}{c - x^4}}$, (iii) $y = -\ln(xe^{-x} + e^{-x} + c)$,

(iv) $y = \tan(e^x + c)$, (v) $y = \frac{1}{2}x^2 e^{3x} + 4e^{3x}$, (vi) $y = 2x^2 \ln x - x + cx^2$.

2. (i) $y = e^{-x}(A \cos 2x + B \sin 2x) + \frac{2}{5}x^2 - \frac{3}{25}x - \frac{14}{125}$, (ii) $y = Ae^{3x} + Be^{-x} - \frac{7}{3}e^{2x}$,

(iii) $y = (Ax + B)e^{-5x} + \frac{42}{841} \cos 2x + \frac{40}{841} \sin 2x$.

3. (i) -0.12061 , (ii) 0.840699 , (iii) 0.887862 .

4. (i) $\frac{16}{3}$, (ii) $\frac{766}{15}$.

5. (i) 6 , (ii) $\frac{3}{56}$.