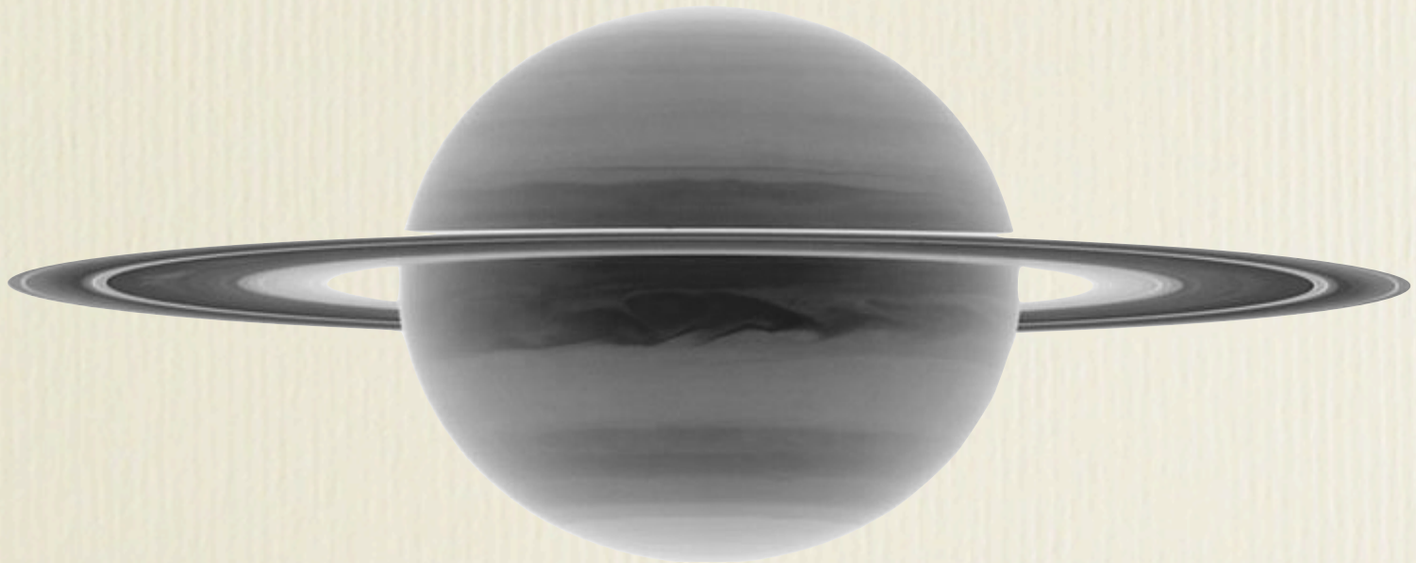
Current Research

- Geometric Structures of Higher Dimensional Black Holes
- In D>4, uniqueness theorems of Hawking et al. are invalid.
- Unusual objects, like 5D black rings, are known to exist.
- Previous results: finding new concentric black ring, and black Saturn solutions in D=5.



Progress in D=10 and D=11 (STFC Advanced Fellowship)

Local vs global properties — conditions for symmetry enhancement

Locally, supersymmetric near-horizon black holes admit parallel spinors ϕ

$$\mathcal{D}_i\phi = 0$$

$\mathcal{D}_i$ is a connection which also incorporates the generalized electromagnetic fluxes of the theory.

Locally, this system of coupled PDEs imposes quite weak conditions on the geometry and fluxes…

We assume global properties of the spatial cross sections of the horizon:

i) All fields are smooth.
ii) The horizon section is smooth and compact, without boundary.

Global Analysis : Generalized Lichnerowicz Theorems & Index Theory

Associated to the connection $\mathcal{D}_i$ there is a generalized horizon Dirac operator.

$$\mathcal{D} = \Gamma^i \mathcal{D}_i$$

I have proven Generalized Lichnerowicz theorems, which state:

$$\mathcal{D}_i \phi = 0 \quad \Longleftrightarrow \quad \mathcal{D}\phi = 0$$

provided that the field equations (e.g. Einstein equations) hold.

So the number of spinors is equal to the number of zero modes of the Dirac operator… which in turn is constrained by the index:

$$\mathrm{Index}(\mathcal{D}) = \dim\Big(\mathrm{Ker}(\mathcal{D})\Big) - \dim\Big(\mathrm{Ker}(\mathcal{D}^\dagger)\Big)$$

In almost all cases the index of the Dirac operator vanishes, so

$$\dim\Big(\mathrm{Ker}(\mathcal{D})\Big) = \dim\Big(\mathrm{Ker}(\mathcal{D}^\dagger)\Big)$$

So the number of zero modes of the adjoint Dirac operator $\mathcal{D}^\dagger$ is equal to the number of zero modes of $\mathcal{D}$.

A further generalized Lichnerowicz theorem for $\mathcal{D}^\dagger$ implies that there is another supercovariant connection $\tilde{\mathcal{D}}_i$, with

$$\Gamma^i\tilde{\mathcal{D}}_i = \mathcal{D}^\dagger$$

such that

$$\tilde{\mathcal{D}}_i\chi = 0 \quad \Longleftrightarrow \quad \mathcal{D}^\dagger\chi = 0$$

So the number of $\mathcal{D}_i$ -parallel and $\tilde{\mathcal{D}}_i$- parallel spinors is equal.

Consequences:

- Supersymmetry is enhanced.
- Extra supersymmetry $\Longrightarrow$ extra symmetry (isometries from spinor bilinears).
- All near horizon geometries in D=11, and D=10 IIA (massive) have a $SL(2,\mathbb{R})$ symmetry.
- Novel (and unexpected) connection between black hole physics and pure mathematics - Index Theory (Atiyah).