Projects/literature reviews suggested by Gianne Derks, 2011-12

Below are some ideas for projects and literature reviews. If you are interested, just come along to my office (36AA04) or send an email to make an appointment to to discuss the projects. If you have some other ideas for a project or literature review, feel free to come and discuss those ideas.

Can you hear the shape of a drum? (project or literature review)

"Can you hear the shape of a drum?" is the title of of an article by Mark Kac in the American Mathematical Monthly 1966. The sound of a drum is associated with its harmonics. By using Helmholz equation, the harmonics can be determined if one knows the shape of a drum. And the question of "hearing the shape of a drum" is asking if the shape of a drum can be found if all harmonics are known. This project/literature review can take several directions. You can look at how the harmonics can be found if the shape is known. And/or you can look at the question of hearing the shape of a drum. In the beginning of the nineties, it is shown that the answer is "no". But you need a non-convex drum. For certain types of convex drums, it can be shown that the harmonics are unique.

A mathematical model related to DNA copying (project)

This project focuses on a mathematical model for the interaction between DNA and RNAP (the polymer essential in DNA reproduction. The model is a PDE, but we will focus on solutions that can be analysed by using ODE techniques. One of the techniques is matching of two of three phase portraits, such that they nicely align at the area of interaction between the DNA and RNAP. This is a simple, but very efficient technique. The project will teach you about those techniques. The project is a combination of analytical and numerical (Matlab) work.

Mathematical modelling of desertfication (project or literature review)

It has been observed that fertile areas can become deserts due to droughts, over-cultivation or similar processes. Once this has happened, it is very hard to get the area back to being fertile again. Some mathematical modelling has been done in the literature and a possible explanation involving bi-stable steady states has been observed. This project/literature review will look at the relevant papers, explore the literature and in case of a project will analyse the models further. Some relevant papers for this project are:

[**1**] Sonia Méfi, Max Rietkerk, Minus van Baalen, and Michel Loreau. Local facilitation, bistability and transitions in arid ecosystems. Theoretical Population Biology **71** (2007), pp 367-379.

[**2**] Sonia Méfi, Max Rietkerk, Conceptión L. Alados, Yolanda Pueyo, Vasilios P. Papanastasis, Ahmed ElAich, and Peter C. de Ruiter. Spatial vegetation patterns and imminent desertification in Mediterranean arid ecosystems. Nature, **449** (2007), pp 213-217.

Bessel functions and Navier-Stokes equations (project or literature review)

The solutions of the differential equation $u''(x) = -k^2 u(x)$, with k some constant in $\mathbb{R}$, are linear combinations of the functions $\sin kx$ and $\cos kx$. Boundary conditions, for example $u(0) = 0 = (1)$, select a subset of the total set of solutions and usually add a condition on k, in this case $k = n\pi$, $n \in \mathbb{N}$. On a square, something similar happens: the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} =$

$-(k^2 + l^2)u$ has as solutions linear combinations of $\sin kx \sin ly$, $\sin kx \cos ly$, $\cos kx \sin ly$ and $\cos kx \cos ly$. Boundary conditions on the sides of the square will give conditions for k and l. However, if we consider the equation on a circular disk, the sine and cosine functions are not very convenient to deal with any boundary conditions on the disk. In this case, the socalled Bessel functions are more convenient. These functions are well-studied and have very nice properties.

For this project or literature review, you will first read through literature to get an overview of some of the properties of Bessel functions. For the project you will continue to apply the Bessel functions to analyse certain solutions of the Navier-Stokes equations. The Navier-Stokes equations describe the motion of fluids. They are quite famous, you can even earn one million dollar by proving that solutions do (not) exist and are (not) uniqueness for the three dimensional version, see `http://www.claymath.org/millennium/Navier-Stokes_Equations/`. This project is less ambitious and will focus on a two-dimensional problem and the influence of boundary conditions. The Bessel functions are used to describe certain solutions. The project will involve a combination of analysis and numerics (Maple or Matlab).

Rossby waves and the spring-swing (literature review or project)

The large sinuous oscillations of the atmospheric flow are called Rossby waves. They are dominant in determining the patterns of weather and climate in middle latitudes, in particular the changeable weather with we are blessed. You can see them any night on the TV weather forecast maps. Rossby waves interact with each other in groups of three, known as resonant triads and, for small amplitude, they are described by the three-wave equations . Their interactions are crucial for determining the distribution of energy in the atmosphere.

A couple of year ago it was discovered that these same equations also govern the dynamics of a simple mechanical system, the elastic pendulum, comprising a heavy mass suspended by a spring. Thus, the motion of a swinging spring gives us information about resonant triads. For a literature review, you can investigate the connection between Rossby waves and the swinging spring. For a project you can some of the solutions and see how solutions can change suddenly if parameters change.

Wavelets (literature review)

The fundamental idea behind wavelets is to analyse according to scale. Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. The idea is similar to the idea behind Fourier series and integrals where sines and cosines are used to represent other functions. However, wavelets are better in taking different scales into account. If we look at a signal with a large "window," we would notice dominant features. Similarly, if we look at a signal with a small "window," we would notice small features. The result in wavelet analysis is to see both the forest and the trees, so to speak. The advantage of wavelets over traditional Fourier is especially noticable when the data or function contains discontinuities and sharp spikes.

Applications of wavelets are widely varied and include astronomy, acoustics, data compression, nuclear engineering, sub-band coding, signal and image processing, neurophysiology, music, magnetic resonance imaging, speech discrimination, optics, fractals, turbulence, earthquake-prediction, radar, human vision, and pure mathematics applications such as solving partial differential equations.

Mathematical theorems and physical experiments(literature review)

It is well-known that mathematics in crucial in describing physics, but it is less known that physical experiments can give ideas and help proving a mathematical theorem. Mark Levi's book " The Mathematical Mechanic: Using Physical Reasoning to Solve Problems" gives some nice illustrations and further references, see `http://press.princeton.edu/titles/8861.html`

Mathematics and Music (literature review)

Literature review based on the book "Music and Mathematics: From Pythagoras to Fractals" by John Fauvel, Raymond Flood, and Robin Wilson, see

`http://www.oup.com/uk/catalogue/?ci=9780199298938`

Graphs and networks (literature review)

Literature review based on the books "Discrete Mathematics, Second Edition" by Norman L. Biggs and "A First Course in Combinatorial Mathematics, Second Edition" by Ian Anderson, see `http://www.oup.com/uk/catalogue/?ci=9780198507185` and `http://www.oup.com/uk/catalogue/?ci=9780198596738`

Elliptic functions and integrals (literature review)

The study of elliptical integrals can be said to start in 1655 when Wallis began to study the arc length of an ellipse. He could derive an integral, but he could not find an expression in elementary functions. The integral was of the type $\int_0^{\pi/2} \sqrt{1-k^2\sin^2\theta}d\theta$. This integral is called a complete elliptic integral of the second kind. A complete elliptic integral of the first kind can be written as $\int_0^{\pi/2} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}$. This integral comes up when one tried to find the period of an pendulum with a large amplitude. Elliptic integrals satisfy all kinds of interesting properties. There are intriguing relations between the algebraic-geometric mean and elliptic integrals.

The incomplete elliptic integrals are the integrals

$$\int_0^{\phi} \sqrt{1-k^2\sin^2\theta}d\theta \quad \text{and} \quad \int_0^{\phi} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}.$$

These integrals can be used to define the socalled elliptic functions. In the limit for $k=0$, the elliptic functions are like sine and cosine functions, while in the limit for $k=1$, they are related to hyperbolic sine and cosine functions. The elliptic functions can also be defined as solutions of special differential equations. Many properties of these functions can be explored in this literature review.

Solitons in optical data transmission (project)

One way of optically transmitting data across a cable uses two (or more) coupled fibre cables. Experiments have shown that if one puts a certain type of signal on one end of the cable, it will go to the other end of this cable and hardly anything happens in the other cable. However, if one puts other types of signals on the cable, the signal will switch to the other cable. This gives a convenient way of sending data consisting of 0 and 1's.

It is an open question how this process exactly works and why certain signals do switch and others don't. A mathematical model for the process is given by a set of partial differential equations, called the "coupled nonlinear Schrodinger equations". The experimentally observed signals behave like socalled solitons (very sharply peaked waves). The soliton solutions of the mathematical model can be described by an ordinary differential equation, which conserves energy and has some extra symmetries.

For this project we would like to start getting a better understanding of the switching process by looking more closely at the soliton solutions. This can be done both in an analytic and/or a numerical way.

Cowboys in a double pendulum (project)

Hamiltonian systems occur often in modelling. They are dynamical systems (ordinary differential equations or partial differential equations) with some extra structure. An example of a Hamiltonian system in a physical model is a system which conserves the energy, like an undamped pendulum or a non-viscous fluid. But in real life the nice structure of the Hamiltonian system is often slightly perturbed. For example, a pendulum does not keep swinging, but slowly gets damped. However, often one can observe behaviour of solutions in the perturbed system, which looks very much like the behaviour of solutions in the purely Hamiltonian system. So features of the Hamiltonian system seem relevant for the perturbed system too.

An example of a Hamiltonian system is a set of two coupled spherical pendulums. A spherical pendulum is a pendulum which can move on a sphere, instead of on a circle only. These coupled pendulums have a family of solutions, called cowboy solutions. A cowboy solution looks like the arm of a cowboy swinging a lasso. It is unknown whether or not these solutions are stable and what happens if dissipation and/or forcing are added to the system. In this project we want to study analytically the stability or instability of the cowboy solutions with a small damping by using the energy and the momentum of the coupled pendulums. A further study of the behaviour of the cowboy solutions under stronger damping and forcing can be done in an analytic and/or numerical way.

Interactive Experiments with Mathematics (project)

Most people outside university do not have a good idea of what mathematics at a university involves. In this project you will make a web-site in which people can meet mathematics interactively and in this way get a better idea what mathematics at a university is.

For the project you will choose a topic in mathematics or statistics which you enjoy doing and which can be presented to someone with some knowledge of mathematics, say an A-level student in a few pages. The best way to get to know mathematics is by doing it by yourself. So it seems to be a good idea that the website will contain some "experiments", where people can interactively play with the topic.

You will gain many skills from this project. You will do some literature research to learn about the mathematical problem; you will learn how to make an effective presentation on the web and you acquire knowledge about webdesign (e.g. html and JAVA applets).

See `http://www.maths.surrey.ac.uk/interactivemaths/` for some webpages produced by previous students (under supervision of various staff members).