Projects/literature reviews suggested by Gianne Derks

Below are some ideas for projects and literature reviews, some of which are in collaboration with other staff members. If you are interested, just come along to my office (32AA04) or send an email (G.Derks@surrey.ac.uk) to make an appointment to to discuss the projects. If you have some other ideas for a project or literature review, feel free to come and discuss those ideas.

Modelling cancer growth and therapies (project)

This project will investigate some of the cancer growth models and effects of drug or other therapies. The project involves ODEs/PDEs, and or data science approaches. You will learn a combination of analytical and numerical skills.

Mathematical Pharmacology (project)

Mathematics pharmacology is the mathematical behinds models used in pharmacology. I have contacts with various pharmaceutical companies and each year, there are projects on drug development that students could work on. Most projects involve ODEs and usually you will learn a combination of analytical and numerical skills.

Dynamics in Game Theory (project or literature review)

This project or literature review builds on the module MAT3046 Game Theory. It looks at an application of Game Theory in modelling biological or economical problems, starting with a paper in the literature. Or it can involve designing an app or programme to illustrate some of the ideas behind dynamic games.

A mathematical model related to DNA copying (project)

This project focuses on a mathematical model for the interaction between DNA and RNAP (the polymer essential in DNA reproduction. The model is a PDE, but we will focus on solutions that can be analysed by using ODE techniques. One of the techniques is matching of two of three phase portraits, such that they nicely align at the area of interaction between the DNA and RNAP. This is a simple, but very efficient technique. You will learn this technique and other skills in this project that will involve a combination of analytical and numerical (Matlab) work.

Mathematical modelling of desertfication (project or literature review)

It has been observed that fertile areas can become deserts due to droughts, over-cultivation or similar processes. Once this has happened, it is very hard to get the area back to being fertile again. Some mathematical modelling has been done in the literature and a possible explanation involving bi-stable steady states has been observed. This project/literature review will look at the relevant papers, explore the literature and in case of a project will analyse the models further. Some relevant papers for this project are: :

[1] Sonia Méfi, Max Rietkerk, Minus van Baalen, and Michel Loreau. Local facilitation, bistability and transitions in arid ecosystems. Theoretical Population Biology **71** (2007), pp 367-379.

[2] Sonia Méfi, Max Rietkerk, Conceptión L. Alados, Yolanda Pueyo, Vasilios P. Papanastasis, Ahmed ElAich, and Peter C. de Ruiter. Spatial vegetation patterns and imminent desertification in Mediterranean arid ecosystems. Nature, **449** (2007), pp 213-217.

Tippe-top (project or literature review)

A tippe top is a kind of top. When a tippe top is spun at a high angular velocity, its handle slowly tilts downwards more and more until it lifts the body of the top off the ground with the stem pointing downward. As the top's spinning rate slows, it loses stability and eventually topples ove

A starting point would be a review paper in SIAM Review 50, p. 323, (2008) `http://dx.doi.org/10.1137/080716177` or the older paper `http://dx.doi.org/10.1137/S0036139992235123`.

Can you hear the shape of a drum? (project or literature review)

"Can you hear the shape of a drum?" is the title of of an article by Mark Kac in the American Mathematical Monthly 1966. The sound of a drum is associated with its harmonics. By using Helmholz equation, the harmonics can be determined if one knows the shape of a drum. And the question of "hearing the shape of a drum" is asking if the shape of a drum can be found if all harmonics are known. This project/literature review can take several directions. You can look at how the harmonics can be found if the shape is known. And/or you can look at the question of hearing the shape of a drum. In the beginning of the nineties, it is shown that the answer is "no". But you need a non-convex drum. For certain types of convex drums, it can be shown that the harmonics are unique.

Bessel functions and Navier-Stokes equations (project or literature review)

The solutions of the differential equation $u''(x) = -k^2u(x)$, with k some constant in $\mathbb{R}$, are linear combinations of the functions $\sin kx$ and $\cos kx$. Boundary conditions, for example $u(0) = 0 = (1)$, select a subset of the total set of solutions and usually add a condition on k, in this case $k = n\pi$, $n \in \mathbb{N}$. On a square, something similar happens: the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -(k^2 + l^2)u$ has as solutions linear combinations of $\sin kx \sin ly$, $\sin kx \cos ly$, $\cos kx \sin ly$ and $\cos kx \cos ly$. Boundary conditions on the sides of the square will give conditions for k and l. However, if we consider the equation on a circular disk, the sine and cosine functions are not very convenient to deal with any boundary conditions on the disk. In this case, the socalled Bessel functions are more convenient. These functions are well-studied and have very nice properties.

For this project or literature review, you will first read through literature to get an overview of some of the properties of Bessel functions. For the project you will continue to apply the Bessel functions to analyse certain solutions of the Navier-Stokes equations. The Navier-Stokes equations describe the motion of fluids. They are quite famous, you can even earn one million dollar by proving that solutions do (not) exist and are (not) uniqueness for the three dimensional version, see `http://www.claymath.org/millennium/Navier-Stokes_Equations/`. This project is less ambitious and will focus on a two-dimensional problem and the influence of boundary conditions. The Bessel functions are used to describe certain solutions. The project will involve a combination of analysis and numerics (Maple or Matlab).

Rossby waves and the spring-swing (literature review or project)

The large sinuous oscillations of the atmospheric flow are called Rossby waves. They are dominant in determining the patterns of weather and climate in middle latitudes, in particular the changeable weather with we are blessed. You can see them any night on the TV weather forecast maps. Rossby waves interact with each other in groups of three, known as resonant triads and, for small amplitude, they are described by the three-wave equations . Their interactions are crucial for determining the distribution of energy in the atmosphere.

A couple of year ago it was discovered that these same equations also govern the dynamics of a simple mechanical system, the elastic pendulum, comprising a heavy mass suspended by a spring. Thus, the motion of a swinging spring gives us information about resonant triads. For a literature review, you can investigate the connection between Rossby waves and the swinging spring. For a project you can some of the solutions and see how solutions can change suddenly if parameters change.

Wavelets (literature review)

The fundamental idea behind wavelets is to analyse according to scale. Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. The idea is similar to the idea behind Fourier series and integrals where sines and cosines are used to represent other functions. However, wavelets are better in taking different scales into account. If we look at a signal with a large "window," we would notice dominant features. Similarly, if we look at a signal with a small "window," we would notice small features. The result in wavelet analysis is to see both the forest and the trees, so to speak. The advantage of wavelets over traditional Fourier is especially noticable when the data or function contains discontinuities and sharp spikes.

Applications of wavelets are widely varied and include astronomy, acoustics, data compression, nuclear engineering, sub-band coding, signal and image processing, neurophysiology, music, magnetic resonance imaging, speech discrimination, optics, fractals, turbulence, earthquake-prediction, radar, human vision, and pure mathematics applications such as solving partial differential equations.

Mathematical theorems and physical experiments(literature review)

It is well-known that mathematics in crucial in describing physics, but it is less known that physical experiments can give ideas and help proving a mathematical theorem. Mark Levi's book " The Mathematical Mechanic: Using Physical Reasoning to Solve Problems" gives some nice illustrations and further references, see `http://press.princeton.edu/titles/8861.html`

Mathematics and Music (literature review)

Literature review based on the book "Music and Mathematics: From Pythagoras to Fractals" by John Fauvel, Raymond Flood, and Robin Wilson, see
`http://www.oup.com/uk/catalogue/?ci=9780199298938`

Elliptic functions and integrals (literature review)

The study of elliptical integrals can be said to start in 1655 when Wallis began to study the arc length of an ellipse. He could derive an integral, but he could not find an expression in elementary functions. The integral was of the type $\int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} d\theta$. This integral is called a complete elliptic integral of the second kind. A complete elliptic integral of the first kind can be written as $\int_0^{\pi/2} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}$. This integral comes up when one tried to find the period of an pendulum with a large amplitude. Elliptic integrals satisfy all kinds of interesting properties. There are intriguing relations between the algebraic-geometric mean and elliptic integrals.

The incomplete elliptic integrals are the integrals

$$\int_0^{\phi} \sqrt{1-k^2\sin^2\theta}d\theta \quad \text{and} \quad \int_0^{\phi} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}.$$

These integrals can be used to define the socalled elliptic functions. In the limit for $k=0$, the elliptic functions are like sine and cosine functions, while in the limit for $k=1$, they are related to hyperbolic sine and cosine functions. The elliptic functions can also be defined as solutions of special differential equations. Many properties of these functions can be explored in this literature review.